FIBONACCI SQUARE DIFFERENCE SLITHER LABELING OF PATH ASSOCIATED GRAPHS

Dr. R. SREENIVASAN

ASSISTANT PROFESSOR, PG AND RESEARCH DEPARTMENT OF MATHEMATICS

AGURCHAND MANMULL JAIN COLLEGE, MEENAMBAKKAM, CHENNAI, INDIA

ABSTRACT: The Fibonacci sequence of numbers are a distinct sequence of positive integers that starts with zero and has applications that extend far and wide. In this article, the Fibonacci numbers are used as labels for graphs Umbrella graph, Comb graph, Double Comb graph, and an attempt has been made to establish Fibonacci Square Difference Slither Labeling for the before-said graphs.

KEYWORDS: Fibonacci labeling, Square Difference Labeling, Slither labeling, Umbrella graph, Comb graph, Double Comb graph

METHODOLOGY: The methodology of applying Fibonacci Square Difference Slither labeling by the organized cataloguing of the vertices using the Fibonacci numbers in the slither arrangement and to verify the applicability of the square difference labeling.

INTRODUCTION: The Fibonacci numbers has significant reach in all spheres of life, especially in the area of trading, physical and biological sciences. The concept of labeling too has gamut of applications in field of technology and science. In this paper, various labeling types have been put together and are tested on path related graphs.

SIGNIFICANT DEFINITONS:

FIBONACCI NUMBERS: Fibonacci numbers are numbers having a unique sequence consisting of non-negative integers such that the numbers are obtained by adding the preceding two numbers.

FIBONACCI LABELING: The labeling of vertices and edges using the Fibonacci numbers is the Fibonacci labeling.

SQUARE DIFFERENCE LABELING: The idea of square sum labeling in graphs by Mitesh J. Patel and G.V. Ghodasara [4] stated as; "A graph G is square summed if there is a leeway to define a bijective map $w:V(G) \rightarrow \{0,1,2,..p-1\}$ such that the induced function $w^*:E(G) \rightarrow \mathbb{N}$ given by $w^*(uv) = (t(u))^2 + (t(v))^2$, for each edge $uv \in E(G)$ is an injective map" is taken as the base for this paper. In a similar way, the square difference is the absolute value of $w^*(uv) = (t(u))^2 - (t(v))^2$, for each edge $uv \in E(G)$.

SLITHER LABELING: The task of allocating numbers to the vertices or edges or both in a graph defines the idea of labeling. The slither labeling is the assignment of numbers in the slither formation.

UMBRELLA GRAPH: An umbrella $U_{m,n}$ graph is a graph that is constructed from a path by joining the path P_n to the central vertex of the fan graph f_m. The number of vertices and edges are given respectively by $m + n$ and $2m + n - 2$. The umbrella graph that is considered here is due to Lavanya. S and Ganesan. V [3]. An illustration of an umbrella graph is given below;

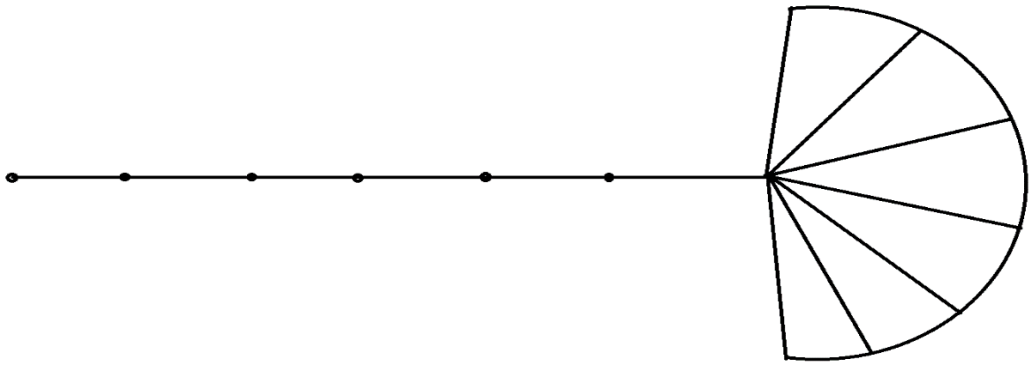


Fig 1. Umbrella Graph $U_{7,7}$

COMB GRPAH: The comb graph represented by CP_n is due to P. Mythili and S. Gokilamani [5], is a graph designed from a path P_n in which the pendant vertices are adjacent with all the vertices in the path. The number of vertices in the graph is 2n and the number of edges in the graph is 2n-1.

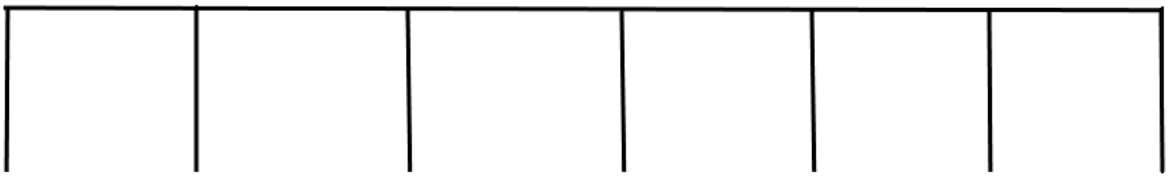


Fig 2. Comb Graph CP_7

MIRROR COMB GRAPH: The comb graph is a special case of a comb graph. The mirror comb graph, MCP_n is the mirror image of the comb graph and attached together such that the path of the comb graph and its mirror image is common.

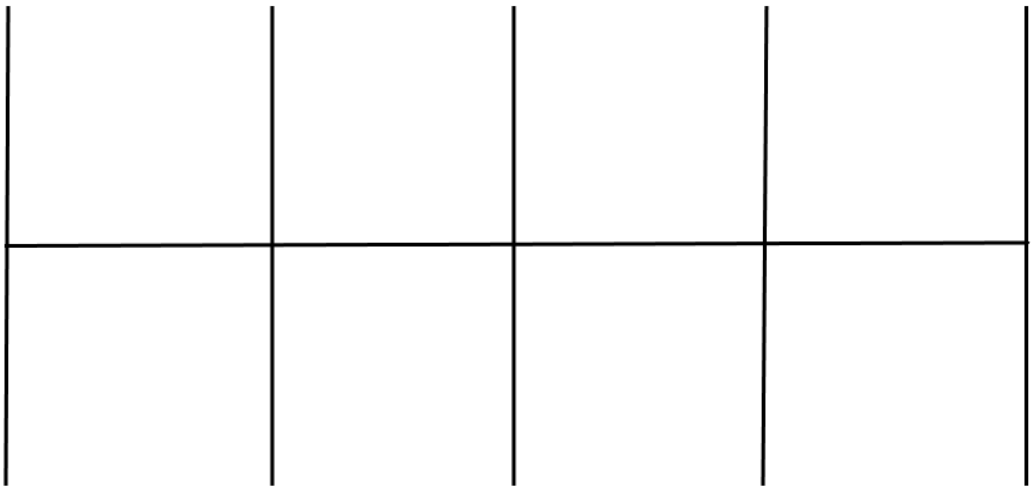


Fig 3. Mirror Comb Graph MCP_5

MAIN RESULTS: The main results of this research article are as follows;

Theorem 1: The Umbrella graph admits Fibonacci Square Difference Slither Labeling.

Proof: : Consider the umbrella graph $U_{m,n}$ on P_n path graph and the fan graph F_m. Label the vertices of the graph in the slither pattern by Fibonacci numbers in a manner that the vertex labels are not repeated. Let m = 4 and n = 5 be arbitrary and the square difference labeling is given as follows;

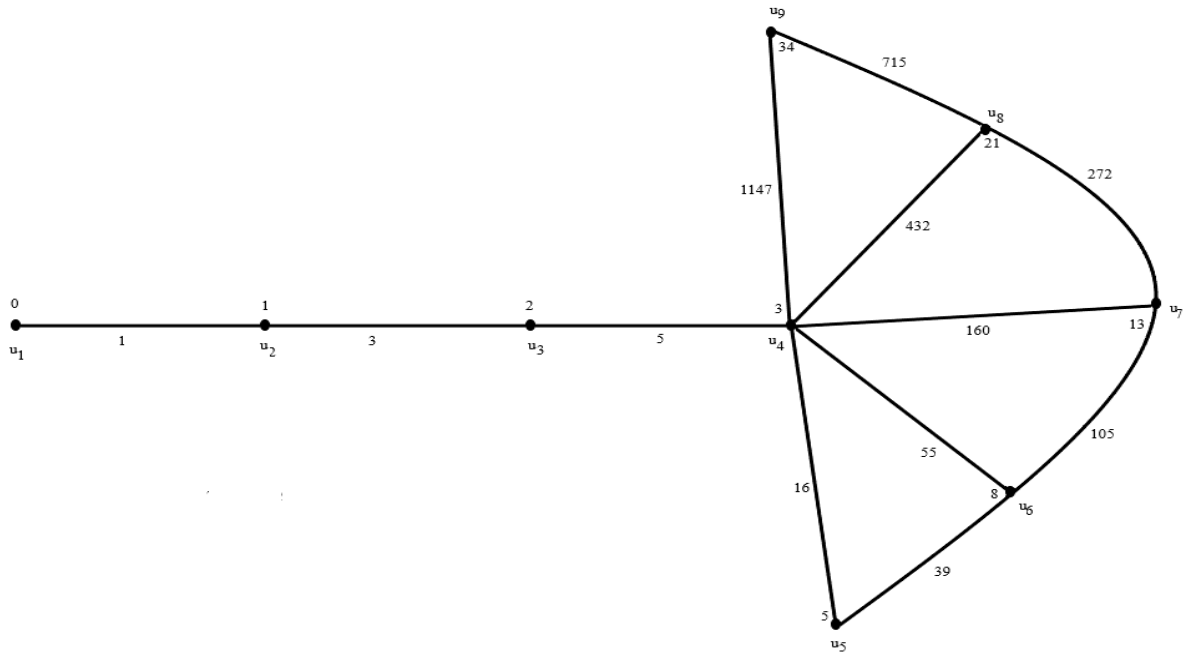


Fig. 4 Fibonacci Square Difference Slither Labeling in $U_{4,5}$.

In the umbrella graph $U_{4,5}$ given above, the vertices are labelled with the Fibonacci numbers from 0 in slither formation and the labels are not repeated. The label of an edge is the absolute value of the square of difference of labels of the vertices that are incident with it and they are all unique as well. Hence the graph is Fibonacci Square Difference Slither Labelled. Since the numbers m = 4 and n = 5 are arbitrary, the proof holds good for all values of $m \geq 2$ and $n \geq 3$. This proves that the graph admits Fibonacci Square Difference Slither Labeling.

Theorem – 2: The Comb graph is a Fibonacci Square Difference Slither Labelled graph.

Proof: Consider the comb graph $CP_{,n}$ that is manipulated from P_n. Assign the vertices of the graph in the slither pattern by Fibonacci numbers in a method that makes vertex labels non-repetitive. Let n = 5 be arbitrary and the square difference labeling is given as follows;

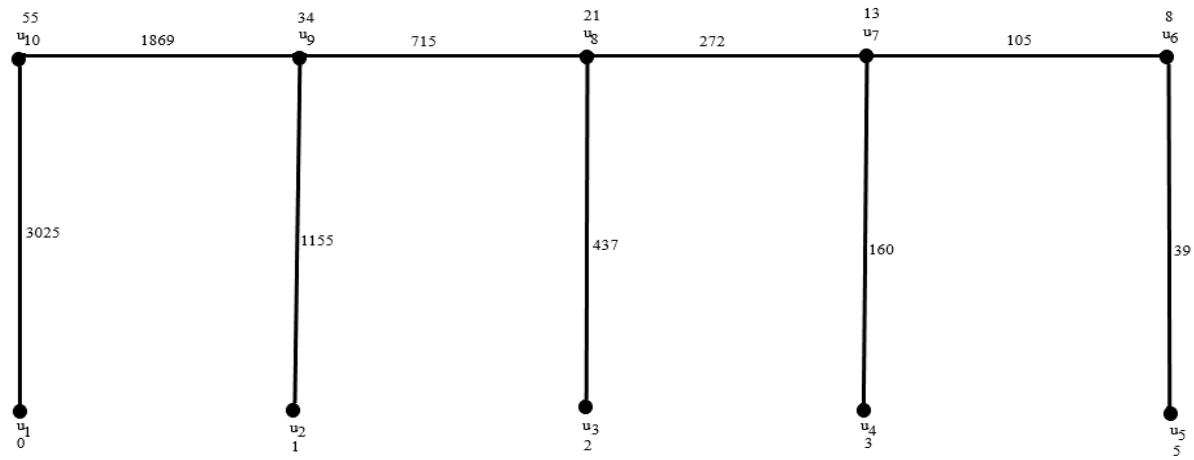


Fig. 5 Fibonacci Square Difference Slither Labeling in CP_5.

In the Comb graph CP_5 given above, the vertices are labelled with the Fibonacci numbers from 0 in slither formation and the labels are not repeated. The label of an edge is the absolute value of the square of difference of labels of the vertices that are incident with it and they are all unique as well. Hence the graph is Fibonacci Square Difference Slither Labelled. Since the number n = 5 is arbitrary, the proof holds good for all values of $n \geq 2$. This proves that the graph admits Fibonacci Square Difference Slither Labeling.

Theorem – 3: The Mirror comb graph is a Fibonacci Square Difference Slither Labelled graph.

Proof: Consider the mirror comb graph $MCP_{,n}$ that is manipulated from P_n. Assign the vertices of the graph in the slither pattern by Fibonacci numbers in a method that makes vertex labels non-repetitive. Let n = 5 be arbitrary and the square difference labeling is given as follows;

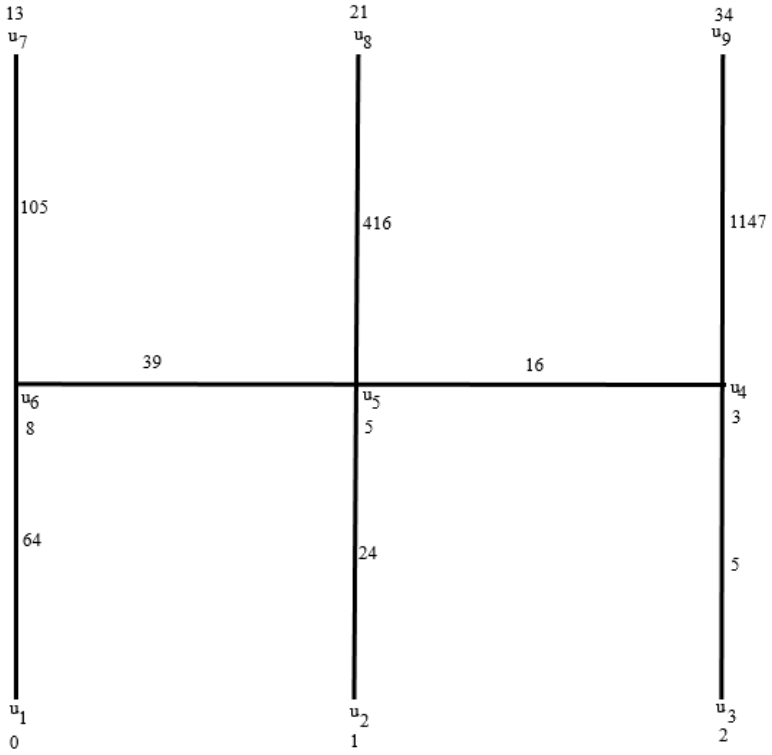


Fig. 6 Fibonacci Square Difference Slither Labeling in MCP_3.

In the Mirror Comb graph MCP_3 given above, the vertices are labelled with the Fibonacci numbers from 0 in slither formation and the labels are not repeated. The label of an edge is the absolute value of the square of difference of labels of the vertices that are incident with it and they are all unique as well. Hence the graph is Fibonacci Square Difference Slither Labelled. Since the number n = 3 is arbitrary, the proof holds good for all values of $n \geq 2$. This proves that the graph admits Fibonacci Square Difference Slither Labeling.

MAJOR FINDING: This research article explains the applicability of Fibonacci Square Difference Slither Labeling to Umbrella graph, Comb graph and the Mirror comb graph.

CONCLUSION: In this research article, three types of labeling are juxtaposed into one single type namely the Fibonacci Square Difference Slither Labeling and is applied to the derivatives of a path graph and it has been proved that the graphs admit the Fibonacci Square Difference Slither Labeling. The same concept can be applied to other graphs as well.

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