



JOURNAL ON COMMUNICATIONS

ISSN:1000-436X

REGISTERED

Scopus®

www.jocs.review

BIPOLAR INTERVAL VALUED INTUITIONISTIC FUZZY NECESSITY OPERATOR

M.Suganya^[1], A.Manonmani^[2]

^[1]Research Scholar, Department of Mathematics, LRG Government Arts college for Women,
Tirupur.

^[2]Assistant Professor, Department of Mathematics, LRG Government Arts college for Women,
Tirupur.

Abstract

In this paper we have introduced the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified its property.

Keyword

Bipolar Interval Valued Intuitionistic Fuzzy Topological Space, Bipolar Interval Valued Intuitionistic Fuzzy Set.

1. Introduction:

Lee introduced the concept of Bipolar fuzzy set. In Bipolar Intuitionistic Fuzzy Topology the membership and non-membership degree of the fuzzy set lies in the range $[0,1]$ and $[-1,0]$ [21]. In this paper we have introduced the Bipolar Interval Valued Intuitionistic Fuzzy necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified that the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topological space.

2. Definition:

Let X be a non-empty set, and let A be a Bipolar interval valued intuitionistic fuzzy set on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X), then the necessity operator on A is defined as

$$i. \quad [A] = \left\{ \left\langle x, \left[\begin{array}{cc} \mu_{AL}^P(x), \mu_{AU}^P(x) \\ \mu_{AL}^{NL}(x), \mu_{AU}^{NU}(x) \end{array} \right], \left[\begin{array}{cc} 1 - \mu_{AL}^P(x), 1 - \mu_{AU}^P(x) \\ 1 + \mu_{AL}^{NU}(x), 1 + \mu_{AU}^{NL}(x) \end{array} \right] \right\rangle \mid x \in X \right\}$$

2.1. Theorem:

Let $(X, \mathbb{I})$ be a Bipolar Interval Valued Intuitionistic Fuzzy Topological Space (BIVIFTS). Based on the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy set A on X , we can also construct several BIVIFTSs on X as

$$\mathbb{I}_N = \{ [A] \mid A \in \mathbb{I} \}$$

i.e., the necessity operator defined in the above definition itself forms a topology.

Proof:

In order to prove the topology we have to prove the following

Let S be a set and $\mathbb{I}$ be a family of bipolar interval valued intuitionistic fuzzy subset of S . The family is called a Bipolar Interval Valued Intuitionistic Fuzzy Topology (BIVIFT) on S if $\mathbb{I}$ satisfies the following axioms

- i. $0_s, 1_s \in \mathbb{I}$
- ii. If $\{A_i; i \in I\} \subseteq \mathbb{I}$, then $\bigcap_{i=1}^{\infty} A_i \in \mathbb{I}$
- iii. If $A_1, A_2, A_3, \dots, A_n \in \mathbb{I}$, then $\bigcap_{i=1}^n A_i \in \mathbb{I}$

Let $A_1, A_2, \dots, A_i$ be Bipolar interval valued intuitionistic fuzzy subsets on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X).

To prove necessity operator is a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X)

- i. obviously $0_s, 1_s \in \mathbb{I}_N$
- ii.

$$A \sqcap B = \left\langle \left[x, \mathbb{I}_{(A \sqcap B)L}^p(x), \mathbb{I}_{(A \sqcap B)U}^p(x) \right], \left[\mathbb{I}_{(A \sqcap B)L}^N(x), \mathbb{I}_{(A \sqcap B)U}^N(x) \right] \right\rangle \mid x \in X$$

$$\left\langle \left[\mathbb{I}_{(A \sqcap B)L}^p(x), \mathbb{I}_{(A \sqcap B)U}^p(x) \right], \left[\mathbb{I}_{(A \sqcap B)L}^N(x), \mathbb{I}_{(A \sqcap B)U}^N(x) \right] \right\rangle$$

where

$$\mathbb{I}_{(A \sqcap B)L}^p(x) = \min \{ \mathbb{I}_{AL}^p(x), \mathbb{I}_{BL}^p(x) \}$$

$$\begin{aligned} \mathbb{I}_{(A \square B)U}^P(x) &= \max \left\{ \mathbb{I}_{AU}^P(x), \mathbb{I}_{BU}^P(x) \right\} \\ \mathbb{I}_{(A \square B)L}^N(x) &= \max \left\{ \mathbb{I}_{AL}^N(x), \mathbb{I}_{BL}^N(x) \right\} \\ \mathbb{I}_{(A \square B)U}^N(x) &= \min \left\{ \mathbb{I}_{AU}^N(x), \mathbb{I}_{BU}^N(x) \right\} \\ \mathbb{I}_{(A \square B)L}^P(x) &= \min \left\{ \mathbb{I}_{AL}^P(x), \mathbb{I}_{BL}^P(x) \right\} \\ \mathbb{I}_{(A \square B)U}^P(x) &= \max \left\{ \mathbb{I}_{AU}^P(x), \mathbb{I}_{BU}^P(x) \right\} \\ \mathbb{I}_{(A \square B)L}^N(x) &= \max \left\{ \mathbb{I}_{AL}^N(x), \mathbb{I}_{BL}^N(x) \right\} \\ \mathbb{I}_{(A \square B)U}^N(x) &= \min \left\{ \mathbb{I}_{AU}^N(x), \mathbb{I}_{BU}^N(x) \right\} \end{aligned}$$

$$\mathbb{I}_{A_1 \square A_2}^P(x) = \left\langle \begin{aligned} &\left[\mathbb{I}_{A_1 \square A_2}^P(x), \mathbb{I}_{A_1 \square A_2}^P(x) \right], \\ &\left[\mathbb{I}_{A_1 \square A_2}^N(x), \mathbb{I}_{A_1 \square A_2}^N(x) \right], \\ &\left[\mathbb{I}_{A_1 \square A_2}^P(x), \mathbb{I}_{A_1 \square A_2}^P(x) \right], \\ &\left[\mathbb{I}_{A_1 \square A_2}^N(x), \mathbb{I}_{A_1 \square A_2}^N(x) \right] \end{aligned} \right\rangle | x \in X$$

where

$$\begin{aligned} \mathbb{I}_{(A_1 \square A_2)L}^P(x) &= \min \left\{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x) \right\} \\ \mathbb{I}_{(A_1 \square A_2)U}^P(x) &= \max \left\{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x) \right\} \\ \mathbb{I}_{(A_1 \square A_2)L}^N(x) &= \max \left\{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x) \right\} \\ \mathbb{I}_{(A_1 \square A_2)U}^N(x) &= \min \left\{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \right\} \\ \mathbb{I}_{(A_1 \square A_2)L}^P(x) &= \min \left\{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x) \right\} \\ \mathbb{I}_{(A_1 \square A_2)U}^P(x) &= \max \left\{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x) \right\} \\ \mathbb{I}_{(A_1 \square A_2)L}^N(x) &= \max \left\{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x) \right\} \\ \mathbb{I}_{(A_1 \square A_2)U}^N(x) &= \min \left\{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \right\} \end{aligned}$$

then

$$\begin{aligned} \mathbb{I}_{(A_1 \square A_2)L}^P(x) &= \min \left\{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x) \right\} \\ \mathbb{I}_{(A_1 \square A_2)U}^P(x) &= \max \left\{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x) \right\} \\ \mathbb{I}_{(A_1 \square A_2)L}^N(x) &= \max \left\{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x) \right\} \end{aligned}$$

$$\begin{aligned}
 \mathbb{Q}_{([A_1 \sqcup [A_2])U}^N(x) &= \min \{ \mathbb{Q}_{A_1U}^N(x), \mathbb{Q}_{A_2U}^N(x) \} \\
 1 \sqcap \mathbb{Q}_{([A_1 \sqcup [A_2])L}^P(x) &= \min \{ 1 \sqcap \mathbb{Q}_{A_1L}^P(x), 1 \sqcap \mathbb{Q}_{A_2L}^P(x) \} \\
 1 \sqcap \mathbb{Q}_{([A_1 \sqcup [A_2])U}^P(x) &= \max \{ 1 \sqcap \mathbb{Q}_{A_1U}^P(x), 1 \sqcap \mathbb{Q}_{A_2U}^P(x) \} \\
 1 \sqcap \mathbb{Q}_{([A_1 \sqcup [A_2])L}^N(x) &= \max \{ 1 \sqcap \mathbb{Q}_{A_1L}^N(x), 1 \sqcap \mathbb{Q}_{A_2L}^N(x) \} \\
 1 \sqcap \mathbb{Q}_{([A_1 \sqcup [A_2])U}^N(x) &= \min \{ 1 \sqcap \mathbb{Q}_{A_1U}^N(x), 1 \sqcap \mathbb{Q}_{A_2U}^N(x) \} \\
 [A_1] \sqcap [A_2] &= \left\langle \begin{aligned} & \left[\mathbb{Q}_{([A_1 \sqcup [A_2])L}^N(x), \mathbb{Q}_{([A_1 \sqcup [A_2])U}^N(x) \right], \\ & \left[1 \sqcap \mathbb{Q}_{([A_1 \sqcup [A_2])L}^P(x), 1 \sqcap \mathbb{Q}_{([A_1 \sqcup [A_2])U}^P(x) \right], \\ & \left[1 \sqcap \mathbb{Q}_{([A_1 \sqcup [A_2])L}^N(x), 1 \sqcap \mathbb{Q}_{([A_1 \sqcup [A_2])U}^N(x) \right] \end{aligned} \right\rangle \mid x \in X \\
 [A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i] &= \left\langle \begin{aligned} & \left[\mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])L}^N(x), \mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])U}^N(x) \right], \\ & \left[1 \sqcap \mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])L}^P(x), 1 \sqcap \mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])U}^P(x) \right], \\ & \left[1 \sqcap \mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])L}^N(x), 1 \sqcap \mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])U}^N(x) \right] \end{aligned} \right\rangle \mid x \in X
 \end{aligned}$$

where

$$\begin{aligned}
 \mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])L}^P(x) &= \min \{ \mathbb{Q}_{A_1L}^P(x), \mathbb{Q}_{A_2L}^P(x), \dots, \mathbb{Q}_{A_iL}^P(x) \} \\
 \mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])U}^P(x) &= \max \{ \mathbb{Q}_{A_1U}^P(x), \mathbb{Q}_{A_2U}^P(x), \dots, \mathbb{Q}_{A_iU}^P(x) \} \\
 \mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])L}^N(x) &= \max \{ \mathbb{Q}_{A_1L}^N(x), \mathbb{Q}_{A_2L}^N(x), \dots, \mathbb{Q}_{A_iL}^N(x) \} \\
 \mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])U}^N(x) &= \min \{ \mathbb{Q}_{A_1U}^N(x), \mathbb{Q}_{A_2U}^N(x), \dots, \mathbb{Q}_{A_iU}^N(x) \} \\
 \mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])L}^P(x) &= \min \{ \mathbb{Q}_{A_1L}^P(x), \mathbb{Q}_{A_2L}^P(x), \dots, \mathbb{Q}_{A_iL}^P(x) \} \\
 \mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])U}^P(x) &= \max \{ \mathbb{Q}_{A_1U}^P(x), \mathbb{Q}_{A_2U}^P(x), \dots, \mathbb{Q}_{A_iU}^P(x) \} \\
 \mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])L}^N(x) &= \max \{ \mathbb{Q}_{A_1L}^N(x), \mathbb{Q}_{A_2L}^N(x), \dots, \mathbb{Q}_{A_iL}^N(x) \} \\
 \mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])U}^N(x) &= \min \{ \mathbb{Q}_{A_1U}^N(x), \mathbb{Q}_{A_2U}^N(x), \dots, \mathbb{Q}_{A_iU}^N(x) \}
 \end{aligned}$$

then

$$\mathbb{Q}_{([A_1 \sqcup [A_2 \sqcup \dots \sqcup A_i])L}^P(x) = \min \{ \mathbb{Q}_{A_1L}^P(x), \mathbb{Q}_{A_2L}^P(x), \dots, \mathbb{Q}_{A_iL}^P(x) \}$$

$$\begin{aligned}
 \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])U}^P(x) &= \max \{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x), \dots, \mathbb{I}_{A_iU}^P(x) \} \\
 \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])L}^N(x) &= \max \{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x), \dots, \mathbb{I}_{A_iL}^N(x) \} \\
 \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])U}^N(x) &= \min \{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x), \dots, \mathbb{I}_{A_iU}^N(x) \} \\
 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])L}^P(x) &= \min \{ 1 \sqcup \mathbb{I}_{A_1L}^P(x), 1 \sqcup \mathbb{I}_{A_2L}^P(x), \dots, 1 \sqcup \mathbb{I}_{A_iL}^P(x) \} \\
 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])U}^P(x) &= \max \{ 1 \sqcup \mathbb{I}_{A_1U}^P(x), 1 \sqcup \mathbb{I}_{A_2U}^P(x), \dots, 1 \sqcup \mathbb{I}_{A_iU}^P(x) \} \\
 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])L}^N(x) &= \max \{ 1 \sqcup \mathbb{I}_{A_1L}^N(x), 1 \sqcup \mathbb{I}_{A_2L}^N(x), \dots, 1 \sqcup \mathbb{I}_{A_iL}^N(x) \} \\
 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])U}^N(x) &= \min \{ 1 \sqcup \mathbb{I}_{A_1U}^N(x), 1 \sqcup \mathbb{I}_{A_2U}^N(x), \dots, 1 \sqcup \mathbb{I}_{A_iU}^N(x) \}
 \end{aligned}$$

$$\left[\begin{array}{c} [A_1] \sqcup [A_2] \sqcup \dots [A_i] \\ \vdots \\ [A_1] \sqcup [A_2] \sqcup \dots [A_i] \end{array} \right] = \left(\begin{array}{c} x, \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])L}^P(x), \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])U}^P(x), \\ \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])L}^N(x), \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])U}^N(x), \\ 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])L}^P(x), 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])U}^P(x), \\ 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])L}^N(x), 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots [A_i])U}^N(x) \end{array} \right) | x \in X$$

Hence the arbitrary union of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

iii.

$$A \sqcup B = \left(\begin{array}{c} [x, \mathbb{I}_{(A \sqcup B)L}^P(x), \mathbb{I}_{(A \sqcup B)U}^P(x)], [\mathbb{I}_{(A \sqcup B)L}^N(x), \mathbb{I}_{(A \sqcup B)U}^N(x)] \\ \mathbb{I}_{(A \sqcup B)L}^P(x), \mathbb{I}_{(A \sqcup B)U}^P(x), [\mathbb{I}_{(A \sqcup B)L}^N(x), \mathbb{I}_{(A \sqcup B)U}^N(x)] \end{array} \right) | x \in X$$

where

$$\begin{aligned}
 \mathbb{I}_{(A \sqcup B)L}^P(x) &= \max \{ \mathbb{I}_{AL}^P(x), \mathbb{I}_{BL}^P(x) \} \\
 \mathbb{I}_{(A \sqcup B)U}^P(x) &= \min \{ \mathbb{I}_{AU}^P(x), \mathbb{I}_{BU}^P(x) \} \\
 \mathbb{I}_{(A \sqcup B)L}^N(x) &= \min \{ \mathbb{I}_{AL}^N(x), \mathbb{I}_{BL}^N(x) \} \\
 \mathbb{I}_{(A \sqcup B)U}^N(x) &= \max \{ \mathbb{I}_{AU}^N(x), \mathbb{I}_{BU}^N(x) \} \\
 \mathbb{I}_{(A \sqcup B)L}^P(x) &= \max \{ \mathbb{I}_{AL}^P(x), \mathbb{I}_{BL}^P(x) \} \\
 \mathbb{I}_{(A \sqcup B)U}^P(x) &= \min \{ \mathbb{I}_{AU}^P(x), \mathbb{I}_{BU}^P(x) \} \\
 \mathbb{I}_{(A \sqcup B)L}^N(x) &= \min \{ \mathbb{I}_{AL}^N(x), \mathbb{I}_{BL}^N(x) \}
 \end{aligned}$$

$$\mathbb{E}_{(A \sqcup B)U}^N(x) = \max \left\{ \mathbb{E}_{AU}^N(x), \mathbb{E}_{BU}^N(x) \right\}$$

then

$$\left(\mathbb{E}_{A_1 \sqcup A_2}^N(x) \right) = \left\langle \begin{array}{l} \left[\mathbb{E}_{A_1 \sqcup A_2}^N(x), \mathbb{E}_{A_1 \sqcup A_2}^N(x) \right], \\ \left[\mathbb{E}_{A_1 \sqcup A_2}^N(x), \mathbb{E}_{A_1 \sqcup A_2}^N(x) \right], \\ \left[\mathbb{E}_{A_1 \sqcup A_2}^N(x), \mathbb{E}_{A_1 \sqcup A_2}^N(x) \right], \\ \left[\mathbb{E}_{A_1 \sqcup A_2}^N(x), \mathbb{E}_{A_1 \sqcup A_2}^N(x) \right] \end{array} \right\rangle | x \in X$$

where

$$\begin{aligned} \mathbb{E}_{A_1 \sqcup A_2}^P(x) &= \max \left\{ \mathbb{E}_{A_1L}^P(x), \mathbb{E}_{A_2L}^P(x) \right\} \\ \mathbb{E}_{A_1 \sqcup A_2}^P(x) &= \min \left\{ \mathbb{E}_{A_1U}^P(x), \mathbb{E}_{A_2U}^P(x) \right\} \\ \mathbb{E}_{A_1 \sqcup A_2}^N(x) &= \min \left\{ \mathbb{E}_{A_1L}^N(x), \mathbb{E}_{A_2L}^N(x) \right\} \\ \mathbb{E}_{A_1 \sqcup A_2}^N(x) &= \max \left\{ \mathbb{E}_{A_1U}^N(x), \mathbb{E}_{A_2U}^N(x) \right\} \\ \mathbb{E}_{A_1 \sqcup A_2}^P(x) &= \max \left\{ \mathbb{E}_{A_1L}^P(x), \mathbb{E}_{A_2L}^P(x) \right\} \\ \mathbb{E}_{A_1 \sqcup A_2}^P(x) &= \min \left\{ \mathbb{E}_{A_1U}^P(x), \mathbb{E}_{A_2U}^P(x) \right\} \\ \mathbb{E}_{A_1 \sqcup A_2}^N(x) &= \min \left\{ \mathbb{E}_{A_1L}^N(x), \mathbb{E}_{A_2L}^N(x) \right\} \\ \mathbb{E}_{A_1 \sqcup A_2}^N(x) &= \max \left\{ \mathbb{E}_{A_1U}^N(x), \mathbb{E}_{A_2U}^N(x) \right\} \end{aligned}$$

then

$$\begin{aligned} \mathbb{E}_{A_1 \sqcup A_2}^P(x) &= \max \left\{ \mathbb{E}_{A_1L}^P(x), \mathbb{E}_{A_2L}^P(x) \right\} \\ \mathbb{E}_{A_1 \sqcup A_2}^P(x) &= \min \left\{ \mathbb{E}_{A_1U}^P(x), \mathbb{E}_{A_2U}^P(x) \right\} \\ \mathbb{E}_{A_1 \sqcup A_2}^N(x) &= \min \left\{ \mathbb{E}_{A_1L}^N(x), \mathbb{E}_{A_2L}^N(x) \right\} \\ \mathbb{E}_{A_1 \sqcup A_2}^N(x) &= \max \left\{ \mathbb{E}_{A_1U}^N(x), \mathbb{E}_{A_2U}^N(x) \right\} \\ 1 \sqcup \mathbb{E}_{A_1 \sqcup A_2}^P(x) &= \max \left\{ 1 \sqcup \mathbb{E}_{A_1L}^P(x), 1 \sqcup \mathbb{E}_{A_2L}^P(x) \right\} \\ 1 \sqcup \mathbb{E}_{A_1 \sqcup A_2}^P(x) &= \min \left\{ 1 \sqcup \mathbb{E}_{A_1U}^P(x), 1 \sqcup \mathbb{E}_{A_2U}^P(x) \right\} \\ 1 \sqcup \mathbb{E}_{A_1 \sqcup A_2}^N(x) &= \min \left\{ 1 \sqcup \mathbb{E}_{A_1L}^N(x), 1 \sqcup \mathbb{E}_{A_2L}^N(x) \right\} \end{aligned}$$

$$\begin{aligned}
 1 \square \square^N_{([A_1 \square [A_2]_U)}(x) &= \max \left\{ 1 \square \square^N_{A_1 U}(x), 1 \square \square^N_{A_2 U}(x) \right\} \\
 \square [A_1] \square [A_2] &= \left\langle \begin{array}{l} x, \square^p_{([A_1 \square [A_2]_L)}(x), \square^p_{([A_1 \square [A_2]_U)}(x),, \\ \square^N_{([A_1 \square [A_2]_L)}(x), \square^N_{([A_1 \square [A_2]_U)}(x),, \\ 1 \square \square^p_{([A_1 \square [A_2]_L)}(x), 1 \square \square^p_{([A_1 \square [A_2]_U)}(x),, \\ 1 \square \square^N_{([A_1 \square [A_2]_L)}(x), 1 \square \square^N_{([A_1 \square [A_2]_U)}(x) \end{array} \right\rangle | x \square X \square^N
 \end{aligned}$$

$$\square [A_1] \square [A_2] \square \dots \square [A_i] = \left\langle \begin{array}{l} x, \square^p_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x), \square^p_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x), \\ \square^N_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x), \square^N_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x), \\ \square^p_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x), \square^p_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x), \\ \square^N_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x), \square^N_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x) \end{array} \right\rangle | x \square X$$

where

$$\begin{aligned}
 \square^p_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x) &= \max \left\{ \square^p_{[A_1]_L}(x), \square^p_{[A_2]_L}(x), \dots, \square^p_{[A_i]_L}(x) \right\} \\
 \square^p_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x) &= \min \left\{ \square^p_{[A_1]_U}(x), \square^p_{[A_2]_U}(x), \dots, \square^p_{[A_i]_U}(x) \right\} \\
 \square^N_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x) &= \min \left\{ \square^N_{[A_1]_L}(x), \square^N_{[A_2]_L}(x), \dots, \square^N_{[A_i]_L}(x) \right\} \\
 \square^N_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x) &= \max \left\{ \square^N_{[A_1]_U}(x), \square^N_{[A_2]_U}(x), \dots, \square^N_{[A_i]_U}(x) \right\} \\
 \square^p_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x) &= \max \left\{ \square^p_{[A_1]_L}(x), \square^p_{[A_2]_L}(x), \dots, \square^p_{[A_i]_L}(x) \right\} \\
 \square^p_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x) &= \min \left\{ \square^p_{[A_1]_U}(x), \square^p_{[A_2]_U}(x), \dots, \square^p_{[A_i]_U}(x) \right\} \\
 \square^N_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x) &= \min \left\{ \square^N_{[A_1]_L}(x), \square^N_{[A_2]_L}(x), \dots, \square^N_{[A_i]_L}(x) \right\} \\
 \square^N_{([A_1 \square [A_2 \square \dots [A_i]_U)}(x) &= \max \left\{ \square^N_{[A_1]_U}(x), \square^N_{[A_2]_U}(x), \dots, \square^N_{[A_i]_U}(x) \right\}
 \end{aligned}$$

then

$$\square^p_{([A_1 \square [A_2 \square \dots [A_i]_L)}(x) = \max \left\{ \square^p_{A_1 L}(x), \square^p_{A_2 L}(x), \dots, \square^p_{A_i L}(x) \right\}$$

$$\begin{aligned}
 \bigcap_{i=1}^n \mathcal{I}_i^p(x) &= \min \left\{ \mathcal{I}_{A_1 U}^p(x), \mathcal{I}_{A_2 U}^p(x), \dots, \mathcal{I}_{A_i U}^p(x) \right\} \\
 \bigcap_{i=1}^n \mathcal{I}_i^N(x) &= \min \left\{ \mathcal{I}_{A_1 L}^N(x), \mathcal{I}_{A_2 L}^N(x), \dots, \mathcal{I}_{A_i L}^N(x) \right\} \\
 \bigcup_{i=1}^n \mathcal{I}_i^p(x) &= \max \left\{ \mathcal{I}_{A_1 U}^p(x), \mathcal{I}_{A_2 U}^p(x), \dots, \mathcal{I}_{A_i U}^p(x) \right\} \\
 \bigcup_{i=1}^n \mathcal{I}_i^N(x) &= \max \left\{ \mathcal{I}_{A_1 L}^N(x), \mathcal{I}_{A_2 L}^N(x), \dots, \mathcal{I}_{A_i L}^N(x) \right\} \\
 \bigcap_{i=1}^n \mathcal{I}_i^p(x) &= \min \left\{ \mathcal{I}_{A_1 U}^p(x), \mathcal{I}_{A_2 U}^p(x), \dots, \mathcal{I}_{A_i U}^p(x) \right\} \\
 \bigcap_{i=1}^n \mathcal{I}_i^N(x) &= \min \left\{ \mathcal{I}_{A_1 L}^N(x), \mathcal{I}_{A_2 L}^N(x), \dots, \mathcal{I}_{A_i L}^N(x) \right\} \\
 \bigcup_{i=1}^n \mathcal{I}_i^p(x) &= \max \left\{ \mathcal{I}_{A_1 U}^p(x), \mathcal{I}_{A_2 U}^p(x), \dots, \mathcal{I}_{A_i U}^p(x) \right\} \\
 \bigcup_{i=1}^n \mathcal{I}_i^N(x) &= \max \left\{ \mathcal{I}_{A_1 L}^N(x), \mathcal{I}_{A_2 L}^N(x), \dots, \mathcal{I}_{A_i L}^N(x) \right\}
 \end{aligned}$$

Hence the finite intersection of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

Hence the Bipolar Interval Valued Intuitionistic Necessity Operators itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

References:

- [1] Andrijevic.D, “Semipreopen sets”, Mat.Vesnic, 38 (1986), 24 -32.
- [2] Azhagappan. M and M.Kamaraj, “Notes on bipolar valued fuzzy rw-closed and bipolar valued fuzzy rw-open sets in bipolar valued fuzzy topological spaces”, International Journal of Mathematical Archive, 7(3), 30-36, 2016.
- [3] Bhattacharya.B., and Lahiri. B.K., “Semi-generalized closed set in topology”, Indian Jour.Math., 29 (1987), 375 -382.
- [4] Chang.C.L., “Fuzzy topological spaces”, JI. Math. Anal. Appl., 24(1968), 182 □190.
- [5] Dontchev.J., “On generalizing semipreopen sets”, Mem. Fac. sci. Kochi. Univ. Ser. A, Math.,16 (1995), 35 - 48.

- [6] Ganguly.S and Saha.S, “A note on fuzzy Semipreopen sets in fuzzy topological spaces”, Fuzzy sets and system, 18 (1986), 83 - 96.
- [7] Indira.R, Arjunan.K and Palaniappan.N, “Notes on interval valued fuzzy rw-closed, interval valued fuzzy rw-open sets in interval valued fuzzy topological space”, International Journal of Computational and Applied Mathematics.,Vol.3, No.1(2013), 23 -38.
- [8] Jeyabalan.R and K. Arjunan, “Notes on interval valued fuzzy generalized semipreclosed sets”, International Journal of Fuzzy Mathematics and Systems, Volume 3, Number 3 (2013), 215 -224.
- [9] Jeyabalan,R and Arjunan.K, “Interval valued fuzzy generalized semi-preclosed mappings”, IOSR Journal of Mathematics, Vol.8, Issue 5(2013), 40 – 47.
- [10] Lee, K.M., “Bipolar-valued fuzzy sets and their operations”, Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand, (2000), 307 -312.
- [11] Levine.N, “Generalized closed sets in topology”, Rend. Circ. Math. Palermo, 19 (1970),89 -96.
- [12] Mondal.T.K., “Topology of interval valued fuzzy sets”, Indian J. Pure Appl.Math. 30, No.1 (1999), 23 - 38.
- [13] Murugan.V, U.Karupiah & M.Marudai, Notes on multi fuzzy rw-closed, multi fuzzy rw-open sets in multi fuzzy topological spaces, Bulletin of Mathematics and Statistics Research, Vo.4, Issue. 1, 2016, 174-179.
- [14] Sabu Sebastian, T.V.Ramakrishnan, Multi fuzzy sets, International Mathematical Forum, 5, no.50 (2010), 2471-2476.
- [15] Saraf.R.K and Khanna.K., “Fuzzy generalized semipreclosed sets”, Jour.Tripura. Math.Soc., 3 (2001), 59 - 68.
- [16] Selvam.R, K.Arjunan & KR.Balasubramanian,”Bipolar interval valued multi fuzzy generalized semipreclosed sets”, JASC Journal, Vol.6, ISS. 5(2019), 2688 -2697.
- [17] Vinoth.S & K.Arjunan, “A study on interval valued intuitionistic fuzzy generalized semipreclosed sets”, International Journal of Fuzzy Mathematics and Systems, Vol. 5, No. 1 (2015), 47 -55.
- [18] Vinoth.S & K.Arjunan, “Interval valued intuitionistic fuzzy generalized semipreclosed mappings”, International Journal of Mathematical Archive, 6(8) (2015), 129 -135.
- [19] Zadeh.L.A., “Fuzzy sets”, Information and control, Vol.8 (1965), 338 -353.

- [20] K., Mohana; Princy R; and F. Smarandache. "An Introduction to Neutrosophic Bipolar Vague Topological Spaces." Neutrosophic Sets and Systems 29, 1 (2020).
- [21] R.Selvam, K. Arjunan And Kr.Balasubramanian “Bipolar Interval Valued Multi Fuzzy Generalized Semipre Continuous Mappings” Volume: 06 Issue: 12 | Dec 2019.