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Ig*-CLOSED SETS IN FUZZY IDEAL TOPOLOGICAL SPACES

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Abstract:

In this paper we introduce the notion of Ig^* -closed sets, Ig^* -open sets in fuzzy ideal topological space and studied some of its basic properties and characterizations.

It shows this class lies between fuzzy closed sets and fuzzy g -closed sets.

Keywords and Phrases : Ig^* -closed sets, Ig^* -open.

1. Introduction

After the introduction of fuzzy sets by Zadeh [18] in 1965 and fuzzy topology by Chang [2] in 1968, several researches were conducted on the generalization of the notions of fuzzy sets and fuzzy topology. The hybridization of fuzzy topology and fuzzy ideal theory was initiated by Mahmoud [6] and Sarkar [12] independently in 1997. They [6, 12] introduced the concept of fuzzy ideal topological spaces as an extension of fuzzy topological spaces and ideal topological spaces.

A nonempty collection of fuzzy sets I of a set X satisfying the conditions:

- (i) if $A \in I$ and $B \leq A$, then $B \in I$ (heredity),
- (ii) if $A \in I$ and $B \in I$ then $A \cup B \in I$ (finite additivity)

is called a fuzzy ideal on X . The triplex (X, τ, I) denotes a fuzzy ideal topological space with a fuzzy ideal I and fuzzy topology τ [12].

The local function for a fuzzy set A of X with respect to τ and I denoted by $A^*(\tau, I)$ (briefly A^*) in a fuzzy ideal topological space (X, τ, I) is the union of all fuzzy points x_β such that if U is a Q -neighbourhood of x_β and $E \in I$ then for at least one point $y \in X$ for which $U(y) + A(y) - 1 > E(y)$ [12]. The $*$ -closure operator of a fuzzy set A denoted by $Cl^*(A)$ in (X, τ, I) defined as $Cl^*(A) = A \cup A^*$. In (X, τ, I) the collection $\tau^*(I)$ is an extension of fuzzy topological space than τ via fuzzy ideal which is constructed by considering the class $\beta = \{U - E : U \in \tau, E \in I\}$ as a base [6,12].

Recently the concepts of fuzzy semi-I-open sets [4], fuzzy α -I-open sets [16], fuzzy γ -I-open sets [3], fuzzy pre-I-open sets [8] and fuzzy δ -I-open sets [17] have been introduced and studied in fuzzy ideal topological spaces. In the present paper we introduce and study the concept of fuzzy I_g^* -closed sets in fuzzy ideal topological spaces which simultaneously generalizes the concept of I_g^* -closed sets [11].

2. Preliminaries

Let X be a nonempty set. A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if the null fuzzy set 0 and the whole fuzzy set 1 belongs to τ and τ is closed with respect to any union and finite intersection. If τ is a fuzzy topology on X , then the pair (X, τ) is called a fuzzy topological space. The members of τ are called fuzzy open sets of X and their complements are called fuzzy closed sets. The closure of a fuzzy set A of X denoted by $Cl(A)$, is the intersection of all fuzzy closed sets which contains A . The interior [2] of a fuzzy set A of X denoted by $Int(A)$ is the union of all fuzzy subsets contained in A . A fuzzy set A of a fuzzy topological space (X, τ) is called fuzzy semi-open if there exists a fuzzy open set U in X such that $U \leq A \leq Cl(U)$ [1]. A fuzzy set A in (X, τ) is said to be quasi-coincident with a fuzzy set B , denoted by AqB , if there exists a point $x \in X$ such that $A(x) + B(x) > 1$ [4]. A fuzzy set V in (X, τ) is called a Q -neighbourhood of a fuzzy point x_β if there exists a fuzzy open set U of X such that $x_\beta q U \leq V$ [4].

Definition 2.1: A fuzzy set A of a fuzzy topological space (X, τ) is called fuzzy generalized closed written as fuzzy g -closed if $Cl(A) \leq O$ whenever $A \leq O$ and O is fuzzy open [14].

Definition 2.2: A fuzzy set A of fuzzy ideal topological space (X, τ, I) is said to be fuzzy $*$ -closed (resp. fuzzy $*$ -dense in itself) if $A^* \leq A$ (resp. $A \leq A^*$) [12].

Definition 2.3: A fuzzy set A of a fuzzy ideal topological space (X, τ, I) is called fuzzy I_g -closed if $A^* \leq U$, whenever $A \leq U$ and U is fuzzy open in X [13].

Lemma 2.1: $A \leq B \Leftrightarrow \bigcap (Aq(1-B))$, for every pair of fuzzy sets A and B of X [9].

3. Fuzzy Ig^* -closed sets

Definition 3.1: A fuzzy set A of a fuzzy ideal topological space (X, τ, I) is called fuzzy Ig^* -closed if $A^* \leq U$, whenever $A \leq U$ and U is fuzzy g -open in X .

Remark 3.1: Every fuzzy $*$ -closed set of a fuzzy ideal topological space (X, τ, I) is fuzzy Ig^* -closed and every fuzzy Ig^* -closed is fuzzy I_g -closed set but the converse may not be true.

Remark 3.2: In a fuzzy ideal topological space (X, τ, I) , I is fuzzy Ig^* -closed for every $A \in I$.

Theorem 3.1: Let (X, τ, I) be a fuzzy ideal topological space. Then A^* is fuzzy Ig^* -closed for every fuzzy set A of X .

Proof: Let A be a fuzzy set of X and U be any fuzzy g -open set of X such that $A^* \leq U$. Since $(A^*)^* \leq A^*$ it follows that $(A^*)^* \leq U$. Hence A^* is fuzzy Ig^* -closed.

Theorem 3.2: Let (X, τ, I) be a fuzzy ideal topological space and A be a fuzzy Ig^* -closed and fuzzy g -open set in X . Then A is fuzzy $*$ -closed.

Proof: Since A is fuzzy g -open and fuzzy Ig^* -closed and $A \leq A$. It follows that $A^* \leq A$ because A is fuzzy Ig^* -closed. Hence $Cl^*(A) = AUA^* \leq A$ and A is fuzzy $*$ -closed.

Theorem 3.3: Let (X, τ, I) be a fuzzy ideal topological space and A be a fuzzy set of X . Then the following are equivalent:

- (i) A is fuzzy Ig^* -closed.
- (ii) $Cl^*(A) \leq U$ whenever $A^* \leq U$ and U is fuzzy g -open in X .
- (iii) $\bigcap (AqF) \Rightarrow \bigcap (Cl^*(A)qF)$ for every fuzzy closed set F of X .
- (iv) $\bigcap (AqF) \Rightarrow \bigcap (A^*qF)$ for every fuzzy closed set F of X .

Proof: (i) $\Rightarrow$ (ii). Let A be a fuzzy Ig^* -closed set in X . Let $A^* \leq U$ where U is fuzzy g -open set in X . Then $A^* \leq U$. Hence $Cl^*(A) = AUA^* \leq U$. Which implies that $Cl^*(A) \leq U$.

(ii) $\Rightarrow$ (i). Let A be a fuzzy set of X . By hypothesis $Cl^*(A) \leq U$. Which implies that $A^* \leq U$. Hence A is fuzzy Ig^* -closed.

(ii) $\Rightarrow$ (iii). Let F be a fuzzy closed set of X and $\neg(AqF)$. Then $1-F$ is fuzzy open in X and by Lemma 2.1, $A \leq 1-F$. Therefore, $Cl^*(A) \leq 1-F$, because A is fuzzy Ig^* -closed. Hence by Lemma 2.1, $\neg(Cl^*(A)qF)$.

(iii) $\Rightarrow$ (ii). Let U be a fuzzy Ig^* -open set of X such that $A^* \leq U$. Then by Lemma 2.1, $\neg(Aq(1-U))$ and $1-U$ is fuzzy closed in X . Therefore by hypothesis $\neg(Cl^*(A)q(1-U))$. Hence, $Cl^*(A) \leq U$.

(i) $\Rightarrow$ (iv). Let F be a fuzzy g -closed set in X such that $\neg(AqF)$. Then $A \leq 1-F$ where $1-F$ is fuzzy g -open. Therefore by (i) $A^* \leq 1-F$. Hence $\neg(A^*qF)$.

(iv) $\Rightarrow$ (i). Let U be a fuzzy closed set in X such that $A \leq U$. Then by Lemma 2.1, $\neg(Aq(1-U))$ and $1-U$ is fuzzy closed in X . Therefore by hypothesis $\neg(A^*q(1-U))$. Hence $A^* \leq U$ and A is fuzzy Ig^* -closed set in X .

Theorem 3.4: Let (X, τ, I) be a fuzzy ideal topological space and A be a fuzzy Ig^* -closed set. Then $xqCl^*(A) \Rightarrow Cl(x)qA$ for any fuzzy point x of X .

Proof: Let $xqCl^*(A)$. If $\neg(Cl(x)qA)$. Then by Lemma 2.1, $A \leq (1-Cl(x))$. And so by Theorem 3.3(ii), $Cl^*(A) \leq (1-Cl(x))$ because $(1-Cl(x))$ is fuzzy g -open set in X . Which implies that $Cl^*(A) \leq (1-x)$. Hence by Theorem 3.3(ii), $\neg(xqCl^*(A))$, which is a contradiction.

Theorem 3.5: Let (X, τ, I) be a fuzzy ideal topological space and A be fuzzy $*$ -dense in itself fuzzy Ig^* -closed set of X . Then A is fuzzy g -closed.

Proof: Let U be a fuzzy open set of X such that $A \leq U$. Since A is fuzzy Ig^* -closed, by Theorem 3.3 (ii), $Cl^*(A) \leq U$. Therefore, $Cl(A) \leq U$, because A is fuzzy $*$ -dense in itself. Hence A is fuzzy g -closed.

Theorem 3.6: Let (X, τ, I) be a fuzzy ideal topological space where $I = \{0\}$ and A be a fuzzy set of X . Then A is fuzzy Ig^* -closed if and only if A is fuzzy g -closed.

Proof: Since $I = \{0\}$, $A^* = Cl(A)$ for each subset A of X . Now the result can be easily proved.

Theorem 3.7: Let (X, τ, I) be a fuzzy ideal topological space and A, B are fuzzy Ig^* -closed sets of X . Then $A \cup B$ is fuzzy Ig^* -closed.

Proof: Let U be a fuzzy g -open set of X such that $A \cup B \leq U$. Then $A \leq U$ and $B \leq U$. Therefore $A^* \leq U$ and $B^* \leq U$ because A and B are fuzzy Ig^* -closed sets of X . Hence $(A \cup B)^* \leq U$ and $A \cup B$ is fuzzy Ig^* -closed.

Remark 3.3: The intersection of two fuzzy Ig^* -closed sets in a fuzzy ideal topological space (X, τ, I) may not be fuzzy Ig^* -closed.

Example 3.1: Let $X = \{a, b\}$ and A, B be two fuzzy sets defined as follows:

$$A(a) = 0.9, \quad A(b) = 0.7$$

$$B(a) = 0.8, \quad B(b) = 0.7$$

$$U(a) = 0.3, \quad U(b) = 0.4$$

Let $\tau = \{0, U, 1\}$ and $I = \{0\}$. Then A and B are fuzzy Ig^* -closed sets in (X, τ, I) but $A \cap B$ is not fuzzy Ig^* -closed.

Theorem 3.8: Let (X, τ, I) be a fuzzy ideal topological space and A, B are fuzzy sets of X such that $A \leq B \leq Cl^*(A)$. If A is fuzzy Ig^* -closed set in X , then B is fuzzy Ig^* -closed.

Proof: Let U be a fuzzy g -open set such that $B \leq U$. Since $A \leq B$ we have $A \leq U$. Hence, $Cl^*(A) \leq U$ because A is fuzzy Ig^* -closed. Now $B \leq Cl^*(A)$ implies that $Cl^*(B) \leq Cl^*(A) \leq U$. Consequently B is fuzzy Ig^* -closed.

Theorem 3.9: Let (X, τ, I) be a fuzzy ideal topological space and A, B are fuzzy sets of X such that $A \leq B \leq A^*$. Then A and B are fuzzy g -closed.

Proof: Obvious.

Theorem 3.10: Let (X, τ, I) be a fuzzy ideal topological space. If A and B are fuzzy subsets of X such that $A \leq B \leq A^*$ and A is fuzzy Ig^* -closed. Then $A^* = B^*$ and B is fuzzy $*$ -open in itself.

Proof: Obvious.

Theorem 3.11: Let (X, τ, I) be a fuzzy ideal topological space and $\mathcal{F}$ be the family of all fuzzy $*$ -closed sets of X . Then $\tau \subset \mathcal{F}$ if and only if every fuzzy set of X is fuzzy Ig^* -closed.

Proof: Necessity. Let $\tau \subset \mathcal{F}$ and U be a fuzzy g -open set in X such that $A^* \leq U$. Now $U \in \tau \Rightarrow U \in \mathcal{F}$. And so $Cl^*(A) \leq Cl^*(U) = U$ and A is fuzzy Ig^* -closed set in X .

Sufficiency. Suppose that every fuzzy set of X is fuzzy Ig^* -closed. Let $U \in \tau$. Since U is fuzzy Ig^* -closed and $U \leq U$, $Cl^*(U) \leq U$. Hence $Cl^*(U) = U$ and $U \in \mathcal{F}$. Therefore $\tau \subset \mathcal{F}$.

Definition 3.2: A fuzzy set A of a fuzzy ideal topological space (X, τ, I) is called fuzzy Ig^* -open if its complement $1-A$ is fuzzy Ig^* -closed.

Remark 3.4: Every fuzzy $*$ -open set in a fuzzy ideal topological space (X, τ, I) is fuzzy Ig^* -open and every fuzzy Ig^* -open is fuzzy I_g -open. But the converse may not be true.

Theorem 3.12: Let (X, τ, I) be a fuzzy ideal topological space and A is fuzzy set of X . Then A is fuzzy Ig^* -open if and only if $F \leq \text{Int}^*(A)$ whenever F is fuzzy g -closed and $F \leq A$.

Proof: Necessity. Let A be fuzzy Ig^* -open and F is fuzzy g -closed set such that $F \leq A$. Then $1-A$ is fuzzy Ig^* -closed, $1-A \leq 1-F$ and $1-F$ is fuzzy g -open in X . Hence $\text{Cl}^*(1-A) \leq (1-F)$. Which implies that $F \leq \text{Int}^*(A)$.

Sufficiency. Let U be a fuzzy g -open set such that $1-A \leq U$. Then $1-U$ is fuzzy g -closed set of X such that $1-U \leq A$. And so by hypothesis, $1-U \leq \text{Int}^*(A)$. Which implies that $\text{Cl}^*(1-A) \leq U$ and $1-A$ is fuzzy Ig^* -closed. Hence A is fuzzy Ig^* -open.

Corollary 3.1: Let (X, τ, I) be a fuzzy ideal topological space and A is fuzzy set of X . Then A is fuzzy Ig^* -open if and only if $F \leq \text{Int}^*(A)$ whenever F is fuzzy closed and $F \leq A$.

Theorem 3.13: Let (X, τ, I) be a fuzzy ideal topological space and A be a fuzzy set of X . If A is fuzzy Ig^* -open and $\text{Int}^*(A) \leq B \leq A$, then B is fuzzy Ig^* -open.

Proof: Let A be fuzzy Ig^* -open in X then $1-A$ is fuzzy Ig^* -closed. Hence $\text{Cl}^*(1-A) \leq (1-A)$ is fuzzy g -open set. Also $\text{Int}^*(A) \leq \text{Int}^*(B) \Rightarrow \text{Cl}^*(1-B) \leq \text{Cl}^*(1-A)$. Hence, B is fuzzy Ig^* -open.

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