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Square Sum Degree Divided By Diameter Energy of Graphs

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Abstract

The square sum degree divided by diameter matrix $\frac{SSD}{diam}(G)$ of a graph G is a square matrix whose (i,j)th entry is $\frac{d_i^2+d_j^2}{diam}$ whenever $i \neq j$ and otherwise zero. where d_i, d_j is the degree of i^{th} and j^{th} vertex of G. In this paper, we define square sum degree divided by diameter energy $E_{\frac{SSD}{diam}}(G)$ as sum of absolute eigenvalues of $\frac{SSD}{diam}(G)$. Also obtained some bounds an $\frac{SSD}{diam}$ eigenvalues and energy.

Keywords: Square sum degree, Diameter, Eigenvalues, Spectrum and Energy

AMS 2010 subject classification: 05C50

1 Introduction

The basics idea of graph theory were born in 1736 with Eulers paper in which he solved the Konigsberg bridge problem.In the last decades graph theory has established itself as a worthwhile mathematical disciplines and there are many applications of graph theory to a wide variety of subjects which include operation research, Physics, Chemistry, Economics, Genetics, Sociology, Engineering etc. We can associate several matrices which record information about

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vertices and how they interconnected. That is we can given an algebraic structure to every graph.Many interesting result can be proved about graphs using matrices and other algebraic properties. The main use of algebraic structure is that we can translate properties of graphs into algebraic properties and then using the results and methods of algebra, to deduce theorems about graphs. We mainly concendrate on energy of graphs which was introduced by I.Gutman in 1978[5].which is having direct connection with total π -electron energy of a molecule in the quantum chemistry as calculated with the Huckel molecular orbital method. Recently several results on energy related with matrices dealing with degree of vertices and distance between vertices have been studied such as distance energy[7, 9], degree sum energy [6], degree exponent energy [11, 10], degree exponent sum energy [8, 3], degree square sum energy[2, 1, 4] etc. In continuation with this, in order to upgrade, we now introduce concept of degree square sum distance square energy of connected graph. The purpose of this paper is to compute square sum degree divided by diameter matrix denoted by SSDDD(G).

2 Square sum degree divided by diameter matrix and its energy

Let G be a connected graph of order n with vertex set $V(G) = (v_1, v_2, ..., v_n)$.We denote by $d(v_i)$ as the degree of a vertex v_i which is the number of edges incident on it and the distance between two vertices v_i and v_j as d_{ij} ,the length of the shortest path joining them. Motivated from previous research, we now define the degree Square sum degree divided by diameter matrix of a connected graph G as,

$$b_{ij} = \begin{cases} \frac{d_i^2+d_j^2}{diam(G)} & \text{if there is a path between } v_i \text{ and } v_j \\ 0 & \text{otherwise} \end{cases}.$$

The square sum degree divided by diameter matrix is a symmetric matrix with eigen values as $\psi_1 \geq \psi_2 \geq \psi_3 \geq .... \geq \psi_p$.

The characteristic polynomial of $\frac{SSD}{diam}(G)$ is given by $det|\psi I - \frac{SSD}{diam}(G)|$.

The Square sum degree divided by diameter energy of the graph G is defined as sum of absolute values of $\psi_i, i = 1, 2, ..., p$.

$$E\frac{SD}{diam}(G) = \sum_{i=1}^{p} |\psi_i|.$$

3 Properties of Square sum degree divided by diameter energy

Theorem 3.1. *If eigen values of* $\frac{SSD}{diam}(G)$ *are* $\psi_1^+ > \psi_2^+ > \cdots > \psi_r^+$*, then*

1. $\sum_{i=1}^{p} \psi_i = 0$ *and*

2. $\sum_{i=1}^{p} \psi_i^2 = 2\sum_{i=1}^{p} \left(\frac{d_i^2 + d_j^2}{diam(G)}\right)^2 = 2\Phi$

where $\Phi = \sum_{i=1}^{p} \left(\frac{d_i^2 + d_j^2}{diam(G)}\right)^2$.

Proof. (1) Since the diagonal entries are zero the sum of leading diagonal entries of $\frac{SSD}{diam}(G)$ is zero .

Hence $\sum_{i=1}^{p} \psi_i = 0$.

(2) The sum of squares of latent roots of $\frac{SSD}{diam}(G)$ is the sum of latent roots of $[\frac{SSD}{diam}(G)]^2$,

$$\sum_{i=1}^{p} \psi_i^2 = \sum_{i=1}^{p}\sum_{j=1}^{n} u_{ij} u_{ji}$$

$$= 0 + 2\sum_{i<j} (u_{ij})^2$$

$$= 2\sum_{i=1}^{p} \left(\frac{d_i^2 + d_j^2}{diam(G)}\right)^2$$

$= 2\Phi$. □

Theorem 3.2. *If* c_0, c_1 *and* c_2 *are the first three coefficients of characteristic polynomial of* $\frac{SSD}{diam}(G)$ *matrix, then*

1. $c_0 = 1$,

2. $c_1 = 0$ *and*

3. $c_2 = -\Phi$.

Proof. (i)By definition, $\Gamma(\psi, x) = det[\psi I - \Phi]$.
Therefore $c_0 = 1$.

(ii) $c_1 = (-1)^1 \times trace(\Gamma) = -1 \times 0 = 0$.

(iii) By definition $c_2 = \sum_{1\le i<j\le p} \begin{vmatrix} u_{ii} & u_{ij} \\ u_{ji} & u_{jj} \end{vmatrix} = \sum_{1\le i<j\le p} (a_{ii}u_{jj} - u_{ij}u_{ji})$

$$= \sum_{1\le i<j\le p} u_{ii}u_{jj} - \sum_{1\le i<j\le p} a_{ij}^2 = 0 - \Phi = -\Phi .$$

□

We have the following bounds for $\frac{SSD}{diam}(G)$ using McClelland's inequalities.

Theorem 3.3. *Let G be a graph with p vertices, then the upper bound for $\frac{SSD}{diam}(G)$ is*

$$E\left(\frac{SSD}{diam}(G)\right) \le \sqrt{2p\Phi}.$$

Proof. Let $\psi_1 \ge \psi_2 \ge \cdots \ge \psi_p$ be the eigen values of $\frac{SSD}{diam}(G)$, then by Using Cauchy-Schwarz inequality we have,

$$\left[\sum_{i=1}^{p} u_i v_i\right]^2 \le \left[\sum_{i=1}^{p} u_i^2\right]\left[\sum_{i=1}^{p} v_i^2\right].$$

Choose $u_i = 1, v_i = |\psi_i|$ and by Theorem 3.1

$$\left[\sum_{i=1} |\psi_i|\right]^2 \le \left[\sum_{i=1} 1\right]\left[\sum_{i=1} |\psi_i|^2\right] = p\sum_{i=1} \psi_i^2$$

$$\left[E\left(\frac{SSD}{diam}(G)\right)\right]^2 \le p2\Phi.$$

Hence

$$E\left(\frac{SSD}{diam}(G)\right) \le \sqrt{2p\Phi}.$$

□

We present the following lower bounds for $E\left(\frac{SSD}{diam}(G)\right)$.

Theorem 3.4. *Let G be a graph with p vertices. If $\tau = \left|\det \frac{SSD}{diam}(G)\right|$ of G,*

then the lower bound is

$$E\left(\frac{SSD}{diam}(G)\right) \ge \sqrt{2\Phi + p(p-1)\tau^{\frac{2}{p}}}.$$

Proof. By definition we have,

$$E\left(\frac{SSD}{diam}(G)\right)^2 = \left(\sum_{i=1}^{p}|\psi_i|\right)^2 = \left(\sum_{i=1}^{p}|\psi_i|\right)\left(\sum_{j=1}^{p}|\psi_j|\right)$$

$$= \sum_{i=1}^{p}|\psi_i|^2 + \sum_{i\neq j}|\psi_i||\psi_j|.$$

From the inequality of arthimetic and geometric means

$$\frac{1}{p(p-1)}\sum_{i\neq j}|\psi_i||\psi_j| \geq \left(\prod_{i\neq j}|\psi_i||\psi_j|\right)^{\frac{1}{p(p-1)}}.$$

Therefore

$$E\left(\frac{SSD}{diam}(G)\right)^2 \geq \sum_{i=1}^{p}|\psi_i|^2 + p(p-1)\left(\prod_{i\neq j}|\psi_i||\psi_j|\right)^{\frac{1}{p(p-1)}}$$

$$\geq \sum_{i=1}^{p}|\psi_i|^2 + p(p-1)\left(\prod_{i=1}^{p}|\psi_i|^{2(p-1)}\right)^{\frac{1}{p(p-1)}}$$

$$= \sum_{i=1}^{p}|\psi_i|^2 + p(p-1)\left|\prod_{i=1}^{p}\psi_i\right|^{\frac{2}{p}}$$

$$= 2\Phi + p(p-1)\tau^{\frac{2}{p}}.$$

Hence

$$E\left(\frac{SSD}{diam}(G)\right) \geq \sqrt{2\Phi + p(p-1)\tau^{\frac{2}{p}}}.$$

□

Theorem 3.5. *Let r_i and s_i, $1 \leq i \leq p$ be positive real numbers with $M_1 = max_{1\leq i\leq p}(r_i)$, $M_2 = max_{1\leq i\leq p}(s_i)$, $m_1 = min_{1\leq i\leq p}(r_i)$, $m_2 = min_{1\leq i\leq n}(s_i)$ then by theorem 90 of [**?**]*

$$\sum_{i=1}^{p} r_i^2 \sum_{i=1}^{p} s_i^2 \leq \frac{1}{4}\left(\sqrt{\frac{M_1M_2}{m_1m_2}} + \sqrt{\frac{m_1m_2}{M_1M_2}}\right)^2 \left(\sum_{i=1}^{p} r_i s_i\right)^2.$$

Theorem 3.6. *For a graph G with p vertices let $|\psi_1|$ and $|\psi_p|$ are the maximum and minimum eigen values among all $|\psi_i|'s$ of $\frac{SSD}{diam}(G)$ respectively, then we have*

$$E\left(\frac{SSD}{diam}(G)\right) \geq \frac{\sqrt{8p\Phi|\psi_1||\psi_n|}}{|\psi_1| + |\psi_p|}.$$

Proof. Consider a graph G with p vertices let $|\psi_1|$ and $|\psi_p|$ are the maximum and minimum eigen values among all $|\psi_i|'s$ of $\frac{SD}{diam}(G)$ respectively.
From theorem 3.5,

$$\sum_{i=1} r_i^2 \sum_{i=1} s_i^2 \le \frac{1}{4}\left[\sqrt{\frac{M_1M_2}{m_1m_2}} + \sqrt{\frac{m_1m_2}{M_1M_2}}\right]^2 \left[\sum_{i=1} r_i s_i\right]^2.$$

Let $r_i = 1$, $s_i = |\zeta_i|$, $M_1M_2 = |\psi_i|$, $m_1m_2 = |\psi_p|$ then

$$\sum_{i=1} 1^2 \sum_{i=1} \psi_i^2 \le \frac{1}{4}\left[\sqrt{\frac{|\psi_1|}{|\psi_p|}} + \sqrt{\frac{|\psi_n|}{|\psi_1|}}\right]^2 \left[\sum_{i=1} 1|\psi_i|\right]^2$$

From theorem 3.1

$$p2\Phi \le \frac{1}{4}\left[\frac{(|\psi_1| + |\psi_p|)^2}{|\psi_1||\psi_n|}\right]\left[E\left(\frac{SSD}{diam}(G)\right)\right]^2,$$

$$\left[E\left(\frac{SSD}{diam}(G)\right)\right]^2 \ge \frac{8p\Phi|\psi_1||\psi_p|}{(|\psi_1| + |\psi_p|)^2}$$

$$E\left(\frac{SSD}{diam}(G)\right) \ge \frac{\sqrt{8p\Phi|\psi_1||\psi_p|}}{|\psi_1| + |\psi_p|}.$$

□

Theorem 3.7. *Let r_i and s_i, $1 \le i \le n$ be non negative real numbers with $M_1 = max_{1\le i\le n}(r_i)$, $M_2 = max_{1\le i\le n}(s_i)$, $m_1 = min_{1\le i\le n}(r_i)$, $m_2 = min_{1\le i\le n}(s_i)$ then by theorem 3.1 of*

$$\sum_{i=1} r_i^2 \sum_{i=1}^{n} s_i^2 - \left[\sum_{i=1}^{n} r_i s_i\right]^2 \le \frac{n^2}{4}(M_1M_2 - m_1m_2)^2.$$

Theorem 3.8. *For a graph G with p vertices, we have*

$$E\left(\frac{SD}{diam}(\Gamma)\right) \ge \sqrt{2p\Phi - \frac{p^2}{4}(|\psi_1| - |\psi_p|)^2}.$$

Proof. Consider a graph G with p vertices let $|\psi_1|$ and $|\psi_p|$ are the maximum and minimum eigen values among all $|\psi_i|'s$ of $\frac{SSD}{diam}(G)$ respectively.
From theorem 3.7,

$$\sum_{i=1}^{p} r_i^2 \sum_{i=1} s_i^2 - \left[\sum_{i=1}^{p} r_i s_i\right]^2 \le \frac{p^2}{4}(M_1M_2 - m_1m_2)^2.$$

Let $r_i = 1$, $s_i = |\psi_i|$, $M_1M_2 = |\psi_i|$, $m_1m_2 = |\psi_p|$, then

$$\sum_{i=1} 1^2 \sum_{i=1}^{p} \psi_i^2 - \left[\sum_{i=1}^{p} 1|\psi_i|\right]^2 \leq \frac{p^2}{4}(|\psi_1| - |\zeta_p|)^2.$$

From theorem 3.1

$$p2\Phi - E\left(\frac{SSD}{diam}(G)\right)^2 \leq \frac{p^2}{4}(|\psi_1| - |\psi_n|)^2,$$

$$E\left(\frac{SSD}{diam}(G)\right) \geq \sqrt{2p\Phi - \frac{p^2}{4}(|\psi_1| - |\psi_p|)^2}.$$

□

Theorem 3.9. *Let r_i and s_i, $1 \leq i \leq p$ be positive real numbers, then by [13]*

$$|p\sum_{i=1}^{p} r_i s_i - \sum_{i=1}^{p} r_i \sum_{i=1}^{p} s_i| \leq \mu(p)(A - a)(B - b)$$

where a, b, A and B are real constants, that for each i, $1 \leq i \leq p$, $a \leq a_i \leq A$ and $b \leq b_i \leq B$. Further, $\mu(p) = p\lfloor\frac{p}{2}\rfloor\left(1 - \frac{1}{p}\lfloor\frac{p}{2}\rfloor\right)$.

Theorem 3.10. *For a graph G with p vertices, we have*

$$E\left(\frac{SSD}{diam}(G)\right) \geq \sqrt{2p\Phi - \mu(p)(|\psi_1| - |\psi_p|)^2}.$$

Proof. Consider a graph G with p vertices let $|\psi_1|$ and $|\psi_p|$ are the maximum and minimum eigen values among all $|\psi_i|'s$ of $\frac{SSD}{diam}(G)$ respectively.
From theorem 3.9,

$$|n\sum_{i=1}^{p} r_i s_i - \sum_{i=1}^{p} r_i \sum_{i=1}^{p} s_i| \leq \mu(p)(A - a)(B - b).$$

Let $r_i = s_i = |\psi_i|$, $A = B = |\psi_1|$, $a = b = |\psi_n|$ then

$$|p\sum_{i=1} |\psi_i|^2 - \left[\sum_{i=1}^{p} |\psi_i|\right]^2| \leq \mu(p)(|\psi_1| - |\psi_p|)(|\psi_1| - |\psi_p|).$$

From theorem 3.1

$$|p2\Phi - E\left(\frac{SSD}{diam}(G)\right)^2| \leq \mu(p)(|\psi_1| - |\psi_p|)^2,$$

$$E\left(\frac{SSD}{diam}(G)\right) \geq \sqrt{2p\Phi - \mu(p)(|\psi_1| - |\psi_p|)^2}.$$

□

4 Square sum degree divided by diameter matrix and its energy for standard graphs

Theorem 4.1. *Let K_p be a complete graph with p vertices, then*

$$E\left(\frac{SSD}{diam}(K_p)\right) = 2p^3 + 12p^2 - 82p + 116.$$

Proof. The complete graph K_p with p-vertices have their square sum degree by diameter matrix as follows

$$\frac{SSD}{diam}(K_p) = \begin{pmatrix} 0 & 2(p-1)^2 & 2(p-1)^2 & \cdots & 2(p-1)^2 \\ 2(p-1)^2 & 0 & 2(p-1)^2 & \cdots & 2(p-1)^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 2(p-1)^2 & 2(p-1)^2 & \cdots & 2(p-1)^2 & 0 \end{pmatrix}.$$

Its characteristic polynomial is,

$[\psi - (18p^2 - 88p + 118)]\,[\psi - (-(2p^2 - 4p + 2))]^{(p-1)} = 0.$

$$Spectra\left(\frac{SSD}{diam}(K_p)\right) = \begin{pmatrix} (18p^2 - 88p + 118) & -(2p^2 - 4p + 2) \\ 1 & p-1 \end{pmatrix}.$$

Therefore

$$E\left(\frac{SSD}{diam}(K_p)\right) = |(18p^2 - 88p + 118)|1 + |-(2p^2 - 4p + 2)|(p-1)$$

$$= 2p^3 + 12p^2 - 82p + 116.$$

□

Theorem 4.2. *Let S_p^0, $p \geq 3$ be a crown graph with $2p$ vertices, then*

$$E\left(\frac{SSD}{diam}(S_p^0)\right) = \frac{4p^3 + 28p^2 - 172p + 236}{3}.$$

Proof. The crown graph S_p^0 with p-vertices has it's square sum degree by diameter matrix as follows

$$\frac{SSD}{diam}(S_p^0) = \begin{pmatrix} 0 & \frac{2(p-1)^2}{3} & \frac{2(p-1)^2}{3} & \cdots & \frac{2(p-1)^2}{3} \\ \frac{2(p-1)^2}{3} & 0 & \frac{2(p-1)^2}{3} & \cdots & \frac{2(p-1)^2}{3} \\ \frac{2(p-1)^2}{3} & \frac{2(p-1)^2}{3} & 0 & \cdots & \frac{2(p-1)^2}{3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{2(p-1)^2}{3} & \frac{2(p-1)^2}{3} & \cdots & \frac{2(p-1)^2}{3} & 0 \end{pmatrix}.$$

Its characteristic polynomial is

$$\left[\psi - \frac{38p^2-180p+238}{3}\right]\left[\psi - \frac{-(2p^2-4p+2)}{3}\right]^{2p-1} = 0$$

$$Spectra_{\frac{SSD}{diam}}(S_p^0) = \begin{pmatrix} \frac{38p^2-180p+238}{3} & \frac{-(2p^2-4p+2)}{3} \\ 1 & 2p-1 \end{pmatrix}.$$

Therefore

$$E_{\frac{SSD}{diam}}(S_p^0) = \left|\frac{38p^2-180p+238}{3}\right|(1) + \left|\frac{-(2p^2-4p+2)}{3}\right|(2p-1)$$

$$= \frac{4p^3+28p^2-172p+236}{3}$$

□

Theorem 4.3. *Let $K_{p\times 2}$ be a cocktail party graph with $2p$ vertices, then*

$$E_{\frac{SSD}{diam}}(K_{p\times 2}) = 8p^3+56p^2-344p+472.$$

Proof. The cocktail party graph $K_{p\times 2}$ with $2p$-vertices has it's square sum degree by diameter matrix as follows

$$\frac{SSD}{diam}(K_{p\times 2}) = \begin{pmatrix} 0 & 4(p-1)^2 & 4(p-1)^2 & \cdots & 4(p-1)^2 \\ 4(p-1)^2 & 0 & 4(p-1)^2 & \cdots & 4(p-1)^2 \\ 4(p-1)^2 & 4(p-1)^2 & 0 & \cdots & 4(p-1)^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 4(p-1)^2 & 4(p-1)^2 & \cdots & 4(p-1)^2 & 0 \end{pmatrix}.$$

Its characteristic polynomial is

$$[\psi - (76p^2-360p+476)]\,[\psi - (4p^2-8p+4)]^{2p-1} = 0$$

$$Spectra_{\frac{SSD}{diam}}(K_{p\times 2}) = \begin{pmatrix} (76p^2-360p+476) & (4p^2-8p+4) \\ 1 & 2p-1 \end{pmatrix}.$$

Therefore

$$E_{\frac{SSD}{diam}}(K_{p\times 2}) = \left|(76p^2-360p+476)\right| \cdot 1 + \left|4p^2-8p+4\right| \cdot (2p-1)$$

$$= 8p^3 + 56p^2 - 344p + 472.$$

□

Theorem 4.4. *Let $K_{p,p}$ be a double star graph with p vertices, then*

$$E_{\frac{SSD}{diam}}(K_{p,p}) = \frac{1}{48}(272p^2 - 632p + 756).$$

Proof. The double star graph $K_{p,p}$ with p-vertices has it's square sum degree by diameter matrix as follows

$$\frac{SSD}{diam}(K_{p,p}) = \begin{pmatrix} 0 & \frac{p^2+1}{3} & \frac{p^2+1}{3} & \frac{p^2+1}{3} & \frac{2p^2}{3} & \frac{p^2+1}{3} & \cdots & \frac{p^2+1}{3} \\ \frac{p^2+1}{3} & 0 & \frac{2}{3} & \frac{2}{3} & \frac{p^2+1}{3} & \frac{2}{3} & \cdots & \frac{2}{3} \\ \frac{p^2+1}{3} & \frac{2}{3} & 0 & \frac{2}{3} & \frac{p^2+1}{3} & \frac{2}{3} & \cdots & \frac{2}{3} \\ \frac{p^2+1}{3} & \frac{2}{3} & \frac{2}{3} & 0 & \frac{p^2+1}{3} & \frac{2}{3} & \cdots & \frac{2}{3} \\ \frac{2p^2}{3} & \frac{p^2+1}{3} & \frac{p^2+1}{3} & \frac{p^2+1}{3} & 0 & \frac{p^2+1}{3} & \cdots & \frac{p^2+1}{3} \\ \frac{p^2+1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{p^2+1}{3} & 0 & \cdots & \frac{2}{3} \\ \vdots & & & & & & \ddots & \vdots \\ \frac{p^2+1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{p^2+1}{3} & \frac{2}{3} & \cdots & 0 \end{pmatrix}.$$

$$\left[\psi - \left(\frac{-(35p^2-129p+162)}{}\right)\right]\left[\psi - \frac{(45p^2-103p+122)}{}\right][\psi - (\frac{-16p^2}{})][\psi - \frac{-16}{}]^{(2p-3)} = 0.$$

$\frac{2}{3}$ ... 0

16 16 24 24

1+ 1+

$\frac{p^2+1}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ $\frac{p^2+1}{3}$

–

Its characteristic polynomial is

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a
m

$-(35p^2-129p+162)$ $(45p^2-103p+122)$ $(-16p^2)$ $\underline{-16}$

1 1 1 $(2p-3)$.

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Therefore

$-(35p^2-129p+162)$ $(45p^2-103p+122)$ $-16p^2$

. 16 . . 16 . . 24 . . 24 .

$$= \frac{1}{48}(272p^2 \quad 632p + 756).$$

□

Theorem 4.5. *Let F_p be a Friendship graph with p vertices, then*

$$E_{\frac{SSD}{diam}}(F_p) = \sqrt{9760p^2 - 40544p + 43792} + 4(2p - 1).$$

Proof. The Friendship graph F_p with $2p+1$ vertices has it's square sum degree by diameter matrix as follows

$$\frac{SSD}{diam}(F_p) = \begin{pmatrix} 0 & 2(p^2+1) & 2(p^2+1) & \cdots & 2(p^2+1) \\ 2(p^2+1) & 0 & 4 & \cdots & 4 \\ 2(p^2+1) & 4 & 0 & \ddots & 4 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 2(p^2+1) & 4 & \cdots & 4 & 0 \end{pmatrix}.$$

Its characteristic polynomial is

$$\left[\psi - \left((8p-4) + \sqrt{9760p^2 - 40544p + 43792}\right)\right]\left[\psi - \left((8p-4) - \sqrt{9760p^2 - 40544p + 43792}\right)\right][\psi + (4)]^{2p-1} = 0$$

$$Spectra_{\frac{SSD}{diam}}(F_p) = \begin{pmatrix} (8p-4) + \sqrt{9760p^2 - 40544p + 43792} & (8p-4) - \sqrt{9760p^2 - 40544p + 43792} & 4 \\ 1 & 1 & 2p-1 \end{pmatrix}.$$

Therefore, $E_{\frac{SSD}{diam}}(F_p) =$

$$\left|(8p-4) + \sqrt{9760p^2 - 40544p + 43792}\right| \cdot 1 + \left|(8p-4) - \sqrt{9760p^2 - 40544p + 43792}\right| \cdot 1 + |4| \cdot (2p-1)$$

$$= \sqrt{9760p^2 - 40544p + 43792} + 4(2p - 1).$$

□

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