



JOURNAL ON COMMUNICATIONS

ISSN:1000-436X

Scopus®

REGISTERED

www.jocs.review

Robust estimation for a Shankar Distribution Based Two Non-Identical Unit Cold Standby System with Priority in Repair

Shivam Tiwari[1], Pramendra Singh Pundir[1*], Rohit Patawa[2],Shubham Mishra[1],Anurag Mishra[1]

[1] Department of Statistics, University of Allahabad, Prayagraj,211002, Uttar Pradesh, India.

[2]Department of Statistics, Agra College, Agra, 282002, Uttar Pradesh, India.

ORCID ID- 0000-0002-1781-7274

Abstract:- This paper studies the characteristics of a two non-identical unit system with a cold standby facility, wherein the time to failure and repair of the operating units are assumed to follow the Shankar distribution. Several measures of system effectiveness, such as reliability, mean time to system failure (MTSF), steady-state availability, and expected profit, are obtained using the regenerative point technique. Sensitivity analysis and relative sensitivity analysis are also carried out to examine the impact of variation in system parameters on these performance measures. At the end, a numerical illustration is presented to validate the proposed system model and to demonstrate its applicability in real-world reliability scenarios.

Keywords: Regenerative point technique, Reliability, Availability, MTSF, Shankar Distribution.

Introduction:-

Reliability plays a pivotal role in assessing whether a system or component can consistently perform at or above a required standard, for a specified duration, and under defined operating conditions. Reliability engineers often deal with systems configured in series and parallel, where the overall performance depends on the arrangement of components. In particular, parallel systems-where the system functions as long as at least one component remains operational-tend to exhibit higher overall reliability compared to any individual component. Consequently, reliability analysis of parallel systems has been extensively investigated in the literature. Recently, there has been a lot of focus on determining the reliability measures of common cause failures in non-identical two/three-unit parallel systems with arbitrarily distributed repair times [1], while other studies have focused on reliability modeling of interconnected subsystems under varying repair and failure policies [10] &[4]. [6]develop a stochastic model for repairable systems with non-identical units, incorporating repairman priority and conditional failure to evaluate system reliability.

A key aspect of repairable systems is the impact of maintenance activities on reliability growth or decay. Depending on the type of repair, systems may be restored to “functioning like new", “failed like old," or fall into an intermediate category that is “superior to the aged state but inferior to original." Such considerations are essential in modeling real-life repairable systems. Earlier studies have addressed repair and redundancy in cold standby configurations, including analysis of replaceable repair facilities [2] and optimization problems involving preventive maintenance and inspection policies [12]. Recently, [5]

perform classical and Bayesian analysis of Weibull parameters in presence of outlier. Also, a novel extension of the Weibull distribution, termed the alpha-power transformed Weibull distribution, is proposed by [3]. This new model demonstrates improved fitting performance compared to the classical Weibull distribution as well as several of its established generalizations & [13] present a reliability and availability model for standby systems incorporating working vacations and component retrials to enhance system performance analysis.

In recent years, considerable attention has been directed toward identifying suitable statistical distributions for modeling lifetime data. [11] provided a comparative evaluation of exponential and Lindley distributions, demonstrating that each has advantages depending on the dataset. [7], [8] & [9] investigated repair models that incorporate waiting times and repair priorities. However, the limitations of these conventional models necessitate the exploration of alternative lifetime distributions that provide better fits across diverse application domains.

Motivated by this, the present study investigates a system in which we have a heterogeneous unit as cold standby, and both failure & repair times are modeled using the Shankar distribution. Various system performance measures such as the MTSF, reliability, steady-state availability & profit are derived. Furthermore, a simulation study along with robustness analysis is carried out to validate the analytical results and to assess the effectiveness of the proposed model in real-world scenarios.

Model description and Assumption:-

We examine a two-unit dissimilar system consisting of unit-1 and unit-2, where unit-1 is assigned operational and repair precedence and is therefore referred to as the priority unit, while unit-2 functions as a cold standby. Each unit can exist in the operative state (G), in the failed state (F), or in an intermediate condition where repair or continued operation is in progress. A repair facility is assumed to be always available whenever a unit fails, and switching between units is considered ideal, i.e., instantaneous and fault-free. The lifetime of the i^{th} Unit (i = 1, 2) is modelled as a r. v.(Random Variable) following the one-parameter Shankar distribution, denoted as Shankar(λ_i), with its associated probability density function (p.d.f.) given by Eq. (1). Similarly, repair times are assumed independent and for each unit also follow a Shankar distribution, denoted as Shankar (μ_i).

$$f_i(t)=\frac{\lambda_i^2}{(1+\lambda_i^2)}(\lambda_i + t)e^{-\lambda_i t} \qquad \lambda_i > 0, t > 0 \tag{1}$$

The system states are defined as follows:

S_0 = System operates with the principal unit active & the supportive unit in cold standby.

S_1= System operates with the supportive unit active, while principal unit undergoes repair after failure.

S_2= System failure state where the principal unit remains under repair and supportive unit has also failed.

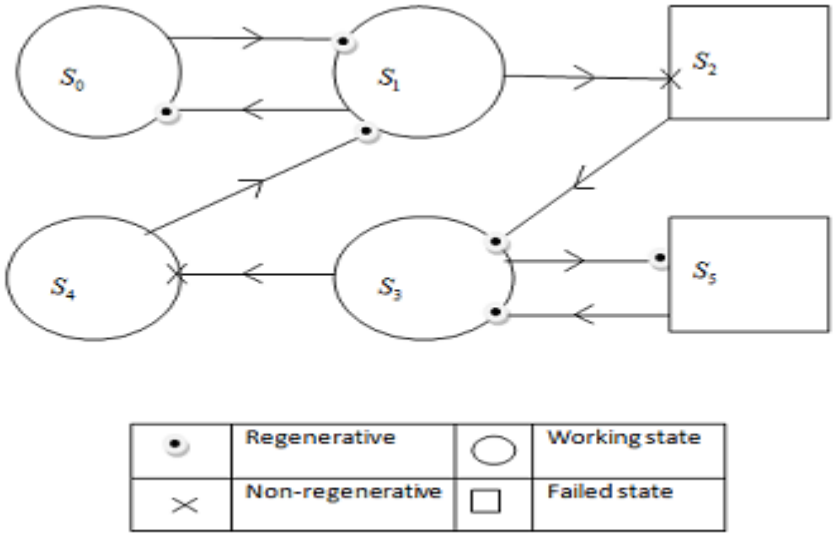
S_3= System operates with the principal unit restored to service after repair, while the supportive unit is under repair.

S_4= System operates with the principal unit active and the supportive unit in cold standby.

S_5=System failure state where the principal unit is under repair and the supportive unit has simultaneously failed.

Transition Characteristics & Mean Sojourn Times:-

$$P=\begin{bmatrix} P_{00} & P_{01} & P_{03} & P_{05} \\ P_{10} & P_{11} & P_{13}^{(2)} & P_{15} \\ P_{30} & P_{31}^{(4)} & P_{33} & P_{35} \\ P_{50} & P_{51} & P_{53} & P_{55} \end{bmatrix} \quad (2)$$



Transition Flow Diagram

Where,

$P_{00} = P_{55} = 1$

$$P_{10} = \left(\frac{\mu_1^2}{1+\mu_1^2}\right)\left(\frac{1}{\mu_1+\lambda_2}\right)\left[\mu_1 + \frac{1}{\mu_1+\lambda_2} + \frac{1}{\mu_1+\lambda_2}\frac{\mu_1\lambda_2}{1+\lambda_2^2} + \frac{\lambda_2}{1+\lambda_2^2}\frac{2}{(\mu_1+\lambda_2)^2}\right]$$

$$P_{35} = \left(\frac{\lambda_1^2}{1+\lambda_1^2}\right)\left(\frac{1}{\mu_2+\lambda_1}\right)\left[\lambda_1 + \frac{1}{\mu_2+\lambda_1} + \frac{1}{\mu_2+\lambda_1}\frac{\mu_2\lambda_1}{1+\mu_2^2} + \frac{\mu_2}{1+\mu_2^2}\frac{2}{(\mu_2+\lambda_1)^2}\right]$$

$P_{13}^{(2)} = 1 - P_{10}$

$P_{31}^{(4)} = 1 - P_{35}$

And other probabilities will be zero.

Mean Sojourn Time of particular state is described as the average amount of time the system spent in one state before transforming to another. Let U_i represents the sojourn time in state S_i, then the mean sojourn time (M_i) is given as-

$$M_i = \int P(U_i > u)\, du \tag{3}$$

The mean sojourn time M_i; i=0, 1, 3 & 5 respectively for different states are outlined as:

$$M_0 = \left(\frac{2+\lambda_1^2}{\lambda_1(1+\lambda_1^2)}\right) \tag{4}$$

$$M_1 = \frac{1}{(\mu_1+\lambda_2)}\left[1 + \left(\frac{\mu_1}{1+\mu_1^2}\right)\left(\frac{1}{\mu_1+\lambda_2}\right) + \left(\frac{1}{\mu_1+\lambda_2}\right)\left(\frac{\lambda_2}{1+\lambda_2^2}\right) + \left(\frac{\mu_1\lambda_2}{(1+\lambda_2^2)(1+\mu_1^2)}\right)\left(\frac{2}{(\mu_1+\lambda_2)^2}\right)\right] \tag{5}$$

$$M_3 = \frac{1}{(\lambda_{1+}+\mu_2)}\left[1 + \left(\frac{\lambda_1}{1+\lambda_1^2}\right)\left(\frac{1}{\lambda_{1+}+\mu_2}\right) + \left(\frac{1}{\lambda_{1+}+\mu_2}\right)\left(\frac{\mu_2}{1+\mu_2^2}\right) + \left(\frac{\mu_2\lambda_1}{(1+\lambda_1^2)(1+\mu_1^2)}\right)\left(\frac{2}{(\lambda_{1+}+\mu_2)^2}\right)\right] \tag{6}$$

$$M_5 = \left(\frac{2+\mu_1^2}{\mu_1(1+\mu_1^2)}\right) \tag{7}$$

Reliability Measure of the System:

To determine $R_i(t)$, at time t, reliability of the system starting from state S_i, is evaluated under the assumption that failed state S_i; i= 2 & 5 act as absorbing state. Employing probabilistic arguments, reliability equations corresponding to two different states can be derived as follows:

$$\boldsymbol{R_0(t) = Y_0(t) + a_{01} \otimes R_1(t)} \tag{8}$$

And
$$R_1(t) = Y_1(t) + a_{10} \otimes R_0(t) \tag{9}$$

Where $\otimes$ used to represent a convolution, and

$$Y_0(t) = \left(1 + \frac{\lambda_1}{1+\lambda_1^2}\, t\right) e^{-\lambda_1 t} \tag{10}$$

$$Y_1(t) = \left(1 + \frac{\lambda_2}{1+\lambda_2^2}\, t\right)\left(1 + \frac{\mu_1}{1+\mu_1^2}\, t\right) e^{(\lambda_1 + \mu_2)t} \tag{11}$$

After applying, the Laplace transformations to (8) & (9) and solving for$R_0^*(s)$, we get,

$$R_0^*(s) = \frac{Y_0^*(s) + a_{01}^*(s) \times Y_1^*(s)}{1 - a_{01}^*(s)a_{10}^*(s)} \tag{12}$$

Once the inverse Laplace transformations of equation (12) are evaluated, the reliability of the system is readily derived for the case where the process starts from state S_0.

When the process is assumed to begin in S_0, MTSF is defined as follows:

$$E(T_0) = \int_0^\infty R_0(t) = \lim_{s\to 0} R_0^*(s) = \frac{M_0 + M_1}{(1 - p_{10})} \tag{13}$$

Let $A_i(t)$ be system's availability at epoch t for state S_i (i=1,2,3,5). Using the basic probabilistic principles, the recursive relations in $A_i(t)$ can be found by:

$$A_0(t) = Y_0(t) + a_{01} \otimes A_1(t) \tag{14}$$

$$A_1(t) = Y_1(t) + a_{10} \otimes A_0(t) + a_{13}^{(2)} \otimes A_3(t) \tag{15}$$

$$A_3(t) = Y_0(t) + a_{13}^{(4)} \otimes A_1(t) + a_{35} \otimes A_5(t) \tag{16}$$

$$A_5(t) = a_{53}(t) \otimes A_3(t) \tag{17}$$

After applying Laplace transformation of (14)-(17), and finding for $A_0^*(s)$, we obtain

$$A_0^*(s) = \frac{D_1(s)}{N_1(s)} = \frac{Y_0^*\left(1 - a_{53}^* a_{35}^* - a_{31}^{(4)*} a_{13}^{(2)*} + a_{01}^* a_{13}^{(2)*}\right) + Y_1^* a_{01}(1 - a_{53}^* a_{35}^*)}{(1 - a_{53}^* a_{35}^*)(1 - a_{01}^* a_{10}^*) - a_{31}^{(4)*} a_{13}^{(2)*}} \tag{18}$$

Taking inverse Laplace transformation of (18), the system's availability is simply obtained. The steady-state operating probability over the long term is determined by

$$A_0 = \lim_{t \to 0} A_0(t) = \lim_{s \to 0} S A_0^*(t) = \frac{p_{31}^{(4)} M_1 + \left(p_{31}^{(4)} + p_{13}^2 p_{35}\right) M_0}{(M_0 + M_5)\left(p_{31}^{(4)} + p_{13}^2 p_{35}\right)} \tag{19}$$

The system's expected uptime for the period of time (0,t) is

$$\tau_{uptime}(t) = \int_0^t A_0(y)dy$$

Let $B_i(t)$ be likelihood of repair when the system first power up, it is in the state S_i (i= 0,1,3,5) and the worker is working on fixing a malfunctioning unit at t. Using the basic probabilistic principles, the recursive relations in $B_i(t)$ can be found by:

$$B_0(t) = a_{01} \otimes B_1(t) \tag{20}$$

$$B_1(t) = Y_5(t) + a_{10} \otimes B_0(t) + a_{13}^{(2)} \otimes B_3(t) \tag{21}$$

$$B_3(t) = Y_3(t) + a_{31}^{(4)} \otimes B_1(t) + a_{35}^{(2)} \otimes B_5(t) \tag{22}$$

$$B_5(t) = Y_5(t) + a_{53}(t) \otimes B_3(t) \tag{23}$$

Where,

$$Y_3(t) = \left(1 + \frac{\lambda_1}{1 + \lambda_1^2} t\right)\left(1 + \frac{\mu_2}{1 + \mu_2^2} t\right) e^{-(\lambda_1 + \mu_2)t} \tag{24}$$

Taking Laplace transformation of (20)-(23), and simplifying for $B_0^*(s)$, we get

$$B_0^*(s) = \frac{D_2(s)}{N_1(s)} = \frac{a_{01}^*\left[a_{13}^{(2)*} Y_3^* + Y_5^*\left(1 - a_{35}^* a_{53}^* + a_{13}^{(2)*} a_{35}^*\right)\right]}{(a_{53}^* a_{35}^*)(1 - a_{01}^* a_{10}^*) - a_{31}^{(4)*} a_{53}^{(2)*}} \tag{25}$$

In the long-term steady-state likelihood that a repairman will be working on fixing malfunctioning equipment is provided by

$$B_0 = \lim_{t\to 0} B_0(t) = \lim_{s\to 0} SB_0^*(t) = \frac{p_{13}^{(2)} M_3 + \left(p_{31}^{(4)} + p_{13}^2 p_{35}\right) M_5}{(M_0 + M_5)\left(p_{31}^{(4)} + p_{13}^2 p_{35}\right)} \quad (26)$$

The repairman's anticipated busy time during the interval (0,t) is

$$\tau_b(t) = \int_0^t B_0(y)dy$$

The system's net profit P(t) for the period between 0 and t is

$$P(t) = q_0 \tau_{uptime} - q_1 \tau_b$$

Where q_0 & q_1 represents the revenue and cost of the system per unit time respectively. Additionally, it is anticipated that both system units would require the same amount of maintenance. In the long run, the system's net anticipated profit is

$$P(t) = q_0 A_0 - q_1 B_0$$

Robustness of the system:-

Criterion robustness is concerned with inference criterion appropriate to our model, examines how sensitive the model is to changes in parameters and the results are based on the local analysis.

1- Sensitivity and Relative Sensitivity Analysis of the Model:

Sensitivity analysis in reliability engineering examines how system performance changes when input parameters vary. Factors such as failure rates, load, environment, and material properties strongly influence reliability measures. Ultimately, it helps optimize performance while ensuring dependable and resilient operation under diverse conditions. The sensitivity & relative sensitivity function for MTSF and system availability are as follow-

$$\rho_k = \frac{\partial(T_0)}{\partial k}\ ,\ \varphi_k = \rho_k\left(\frac{k}{T_0}\right) \& \rho_k = \frac{\partial(A_0)}{\partial k}\ ,\ \varphi_k = \rho_k\left(\frac{k}{A_0}\right)$$

Where, k= $\lambda_1, \lambda_2, \mu_1$& μ_2

The fixed values of the all parameters are 0.01.

Results:-

From the previously derived transition probability relations, the sojourn time corresponding to the system can be determined from Equations.(4 -7), we have obtained the MTSF, steady state availability, and system profit for different parameter values.

Table 1: Values of MTSF function for distinct values of μ_1 respect to λ_1

λ_1	μ_1=2.31	μ_1=1.35	μ_1=0.80
0.01	17604.69	6205.78	2308.80
0.02	8822.79	3117.46	1164.91
0.03	5894.90	2087.81	783.53
0.04	4430.53	1572.84	592.79
0.05	3551.55	1263.73	478.30
0.06	2965.28	1057.55	401.93
0.07	2546.27	910.20	347.35
0.08	2231.80	799.61	306.39
0.09	1987.02	713.53	274.51
0.1	1791.03	644.61	248.98

Table 2: Values of steady-state availability for distinct values of μ_1 respect to λ_1

λ_1	μ_1=2.31	μ_1=1.35	μ_1=0.80
0.01	0.9999716	0.9998496	0.9992657
0.02	0.9999402	0.9996832	0.998457
0.03	0.9999042	0.9994918	0.9975305
0.04	0.9998624	0.9992702	0.9964631
0.05	0.9998142	0.9990144	0.9952406
0.06	0.9997588	0.9987214	0.9938535
0.07	0.9996956	0.9983886	0.9922952
0.08	0.9996242	0.9980135	0.9905612
0.09	0.9995438	0.997594	0.9886483
0.1	0.9994541	0.9971281	0.9865546

Table 3: Values of steady-state profit for distinct values of μ_1 respect to λ_1

λ_1	μ_1=2.31	μ_1=1.35	μ_1=0.80
0.01	99.96078	99.90276	99.74035
0.02	99.92105	99.80375	99.47482

0.03	99.88049	99.70165	99.19821
0.04	99.83892	99.59573	98.90792
0.05	99.79621	99.48554	98.60250
0.06	99.75227	99.37075	98.28106
0.07	99.70703	99.25109	97.94306
0.08	99.66042	99.12633	97.58816
0.09	99.61238	98.99628	97.21622
0.1	99.56285	98.86075	96.82719

Table 4: Values of Sensitivity & Relative Sensitivity of T_0 & A_0 for distinct values of λ_1

λ_1	For T_0		For A_0	
	ρ_k	φ_k	ρ_k	φ_k
0.01	-40001.9994	-0.6154532	-25.001944	0.200016664
0.02	-10001.9976	-0.4445925	-18.999094	0.116016786
0.03	-4446.4391	-0.3480548	-12.457672	0.067061846
0.04	-2501.9904	-0.2860234	-8.295354	0.041057101
0.05	-1601.9851	-0.2428131	-5.746592	0.026606038
0.06	-1113.0896	-0.2109942	-4.141547	0.018097042
0.07	-818.2974	-0.1865925	-3.090349	0.012813693
0.08	-626.962	-0.1672897	-2.375146	0.009379826
0.09	-495.7792	-0.1516423	-1.871669	0.007059733
0.1	-401.941	-0.1387046	-1.506534	0.005439612

Table 5: Values of Sensitivity & Relative Sensitivity of T_0 & A_0 for distinct values of μ_1

μ_1	For T_0		For A_0	
	ρ_k	φ_k	ρ_k	φ_k
0.01	36249.25	0.5577151	16.66805529	0.200016664
0.02	50628.05	0.9326061	5.40281839	0.116016786
0.03	64257.91	1.161035	2.15992088	0.067061846
0.04	77769.29	1.3122696	1.00711353	0.041057101
0.05	91276.69	1.4192176	0.52601763	0.026606038
0.06	104819.9	1.4987894	0.29936325	0.018097042
0.07	118420.17	1.5604103	0.18211444	0.012813693
0.08	132092.63	1.6097013	0.11681857	0.009379826
0.09	145849.93	1.6502046	0.07822954	0.007059733
0.1	159703.51	1.6842548	0.0542847	0.005439612

Fig 2- Behaviour of MTSF for different values of μ_1 with respect to λ_1

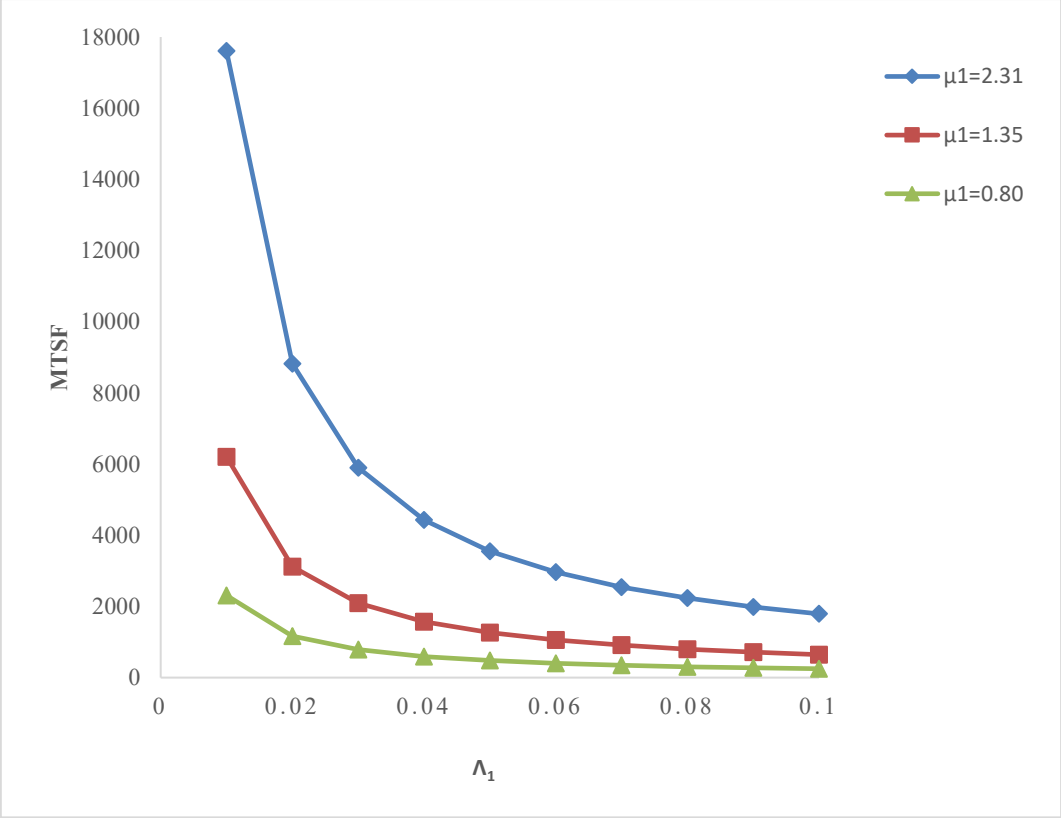


Fig-3 Behaviour of Availability for different values of μ_1 with respect to λ_1

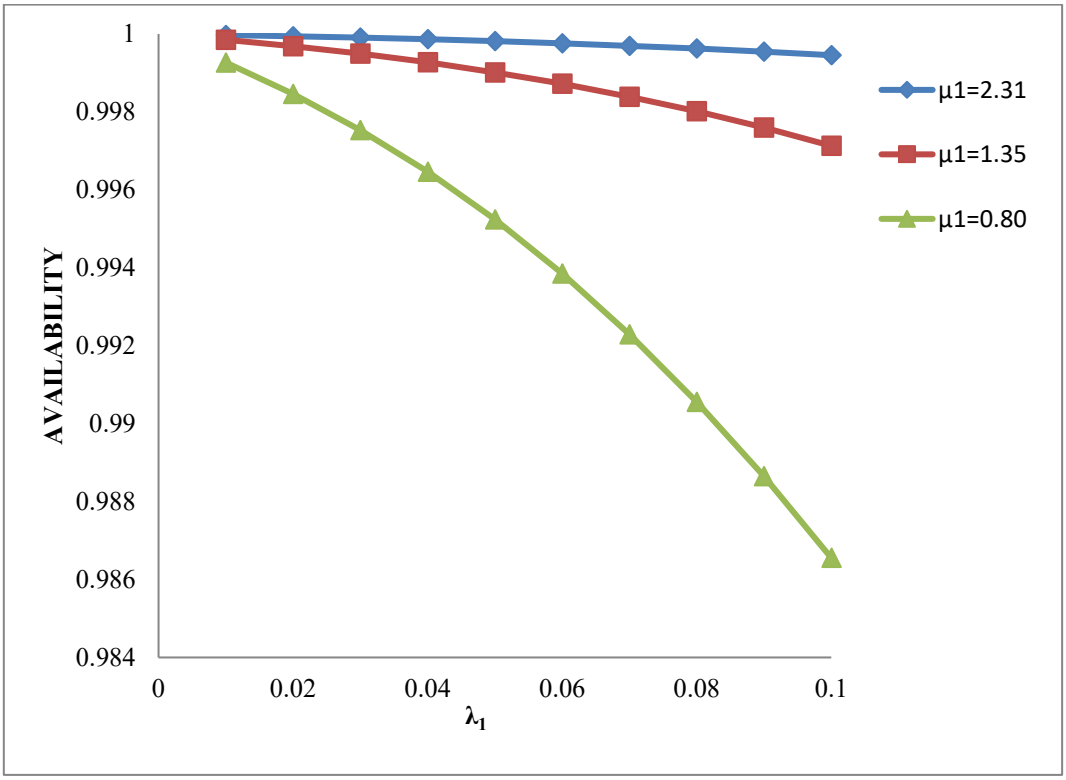
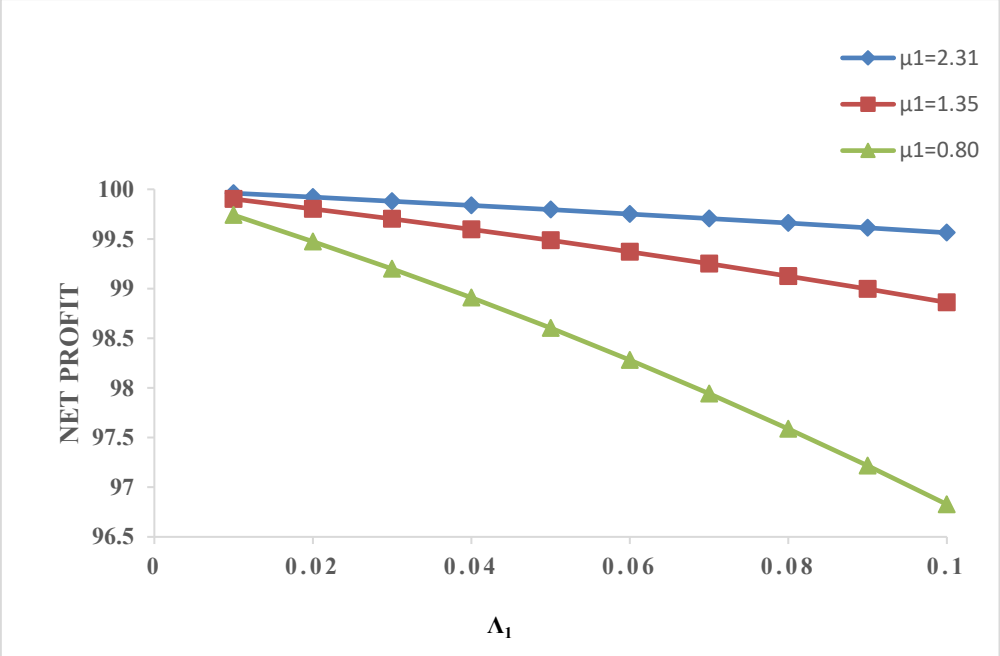


Fig-4 **Behaviour of steady-state profit for different values of μ_1 with respect to λ_1**



The tabulated results confirm that system reliability improves with higher repair rates and deteriorates with higher failure rates. These numerical outcomes provide quantitative evidence to support the importance of optimizing both design (minimizing λ_1) and maintenance strategies (maximizing μ_1). Furthermore, the steep reduction in MTSF at lower μ_1 values emphasizes the practical need for skilled manpower and efficient repair facilities in real-world applications.

Table 1 illustrates the variation of the MTSF with respect to different values of the λ_1 and μ_1. It is evident that with the increase in λ_1, the MTSF decreases consistently, indicates that higher value of λ_1 significantly reduce system reliability (at $\mu_1 = 2.31$, the MTSF decreases from 17,604.69 at $\lambda_1 = 0.01$ to 1,791.03 at $\lambda_1 = 0.1$), while with the increase in μ_1, the system reliability improves (at $\lambda_1 = 0.05$, the MTSF is 3,551.55 for $\mu_1 = 2.31$, but only 478.30 for $\mu_1 = 0.80$). This shows that system reliability improves with increase in repair rates and deteriorates with increasing failure rates, illustrating the crucial role of reliable maintenance strategies in achieving increased operational availability.

Table 2 shows the availability of the system with the change in λ_1 and μ_1. The results indicate that availability of the system under modeled condition decreases as λ_1 increases, regardless of the value of μ_1. For example, when $\mu_1 = 2.31$, availability decreases from 0.99997 at $\lambda_1 = 0.01$ to 0.99945 at $\lambda_1 = 0.1$. Similarly, with the decrease in μ_1, the availability of the system decreases. For example, at $\lambda_1 = 0.01$, availability decreases from 0.99997 for $\mu_1 = 2.31$ to 0.99926 for $\mu_1 = 0.80$, which indicates that, the system become less available and reliable for operation when repairs take longer.

Table 3 tells about the steady-state profit of the system for different values of λ_1 & μ_1, under the modeled assumption. It is clear that as increase in λ_1, the profit decreases, indicates that increase in failure rate, the operational efficiency of the system reduce. For example, when μ_1 = 2.31, the profit varies from 99.96 at λ_1 = 0.01 to 99.56 at λ_1 = 0.1. Similarly, It is clear that as decrease in μ_1, the profit decreases, indicates that increase in repair rate improves the profit values. For instance, at λ_1 = 0.05, the profit varies from 99.79 for μ_1 = 2.31 to 98.60 for μ_1 = 0.80.

Table 4 & 5 presents the ρ_k and φ_k values of MTSF and steady-state availability w. r. t. variations in the λ_1 and μ_1. The results clearly reveal that both sensitivity and relative sensitivity values are negative for all cases, indicating an inverse relationship between λ_1 and the measures of system performance. In other words, an increase in failure rate consistently reduces both T_0 and A_0. While the results show the values of T_0 and A_0 decline as μ_1 increases. This inverse relationship indicates higher service rates reduce availability responsiveness while moderating performance growth. Thus, increasing μ_1 consistently lowers A_0 sensitivity and stabilizesT_0, reflecting reduced system performance impact overall.

Sensitivity with respect to λ_1:-

Table 4 presents the values of ρ_k and φ_k of T_0 and A_0 for distinct values of λ_1.

1- For T_0, the values of ρ_k are highly negative at lower values of λ_1, starting from -40001.9994 at λ_1=0.01. As λ_1 increases, the magnitude of sensitivity decreases significantly, reaching -401.941 at λ_1=0.1 and the values of φ_k also shows a decreasing trend, moving from -0.6154532 at λ_1=0.01 to -0.1387046 at λ_1=0.1.

2- For A_0, the ρ_k are comparatively smaller, beginning at -25.001944 for λ_1=0.01 and reducing to -1.506534 at λ_1=0.1 and the values of φ_k decreases steadily from 0.200016664 to 0.005439612 across the same range of λ_1.

Sensitivity with respect to μ_1:-

Table 5 shows the values of ρ_k and φ_k for T_0 and A_0 for distinct values of μ_1.

(i) For T_0, ρ_k increase sharply with μ_1, starting from 36249.25 at μ_1=0.01 and rising to 159703.51 at μ_1=0.1 & the values of φ_k for T_0 also increases consistently, from 0.5577151 to 1.6842548.

(ii) For A_0, the ρ_k decrease as μ_1 increases, beginning at 16.66805529 for μ_1=0.01 and dropping to 0.0542847 at μ_1=0.1 and similarly, the values of φ_k for A_0 declines from 0.200016664 to 0.005439612.

Conclusion:-

This study has examined the reliability characteristics of a repairable system by evaluating the MTSF, steady-state availability, profit, and sensitivity measures under varying values of the failure rate λ_1 and repair rate μ_1. The numerical results presented in tables 1-3 demonstrate

that MTSF decreases consistently as λ_1 increases, while higher values of μ_1 significantly extend system life. Availability remains close to unity across all parameter ranges, but shows a gradual decline with increasing failure rate, particularly when repair rates are low. Profit values follow a similar trend, decreasing steadily with higher λ_1, though improved repair rates mitigate this reduction and sustain economic performance.

The sensitivity analysis (Tables 4 & 5) provides deeper insight into system responsiveness. For T_0, sensitivity values are negative with respect to λ_1, indicating that increases in failure rate reduce system performance. Conversely, sensitivity values are positive with respect to μ_1, confirming that improvements in repair rate enhance reliability. Relative sensitivity values for availability remain positive but comparatively small, suggesting that availability is less proportionally affected than MTSF.

Finally, the findings highlight a dual behavior i.e. while increasing failure rates diminish reliability, availability, and profit; higher repair rates counterbalance these effects and sustain system efficiency. The sensitivity analysis confirms that T_0 is more strongly influenced by the changes in values of parameter than A_0, underlining its importance in system evaluation. These results emphasize the critical role of effective repair strategies in maintaining high availability and profitability, and demonstrate that sensitivity measures can serve as a valuable tool for optimizing system parameters to ensure long-term operational success.

References:-

1. Al Rahbi, Y. Reliability Analysis of Production in an Engineering System.
2. Cao, J., & Wu, Y. (1989). Reliability analysis of a two-unit cold standby system with a replaceable repair facility. *Microelectronics Reliability*, *29*(2), 145-150.
3. Dey, S., Sharma, V. K., & Mesfioui, M. (2017). A new extension of Weibull distribution with application to lifetime data. *Annals of Data Science*, *4*(1), 31-61.
4. Ghasemi, A., Yacout, S., & Ouali, M. S. (2009). Evaluating the reliability function and the mean residual life for equipment with unobservable states. *IEEE Transactions on Reliability*, *59*(1), 45-54.
5. Gupta, P. K., & Singh, A. K. (2017). Classical and Bayesian estimation of Weibull distribution in presence of outliers. *Cogent Mathematics*, *4*(1), 1300975.
6. Kumar, N., Malik, S. C., & Nandal, N. (2022). Stochastic analysis of a repairable system of non-identical units with priority and conditional failure of repairman. *Reliability: Theory & Applications*, *17*(1 (67)), 123-133.
7. Pundir, P. S., Patawa, R., & Gupta, P. K. (2021). Analysis of two non-identical unit cold standby system in presence of prior information. *American Journal of Mathematical and Management Sciences*, *40*(4), 320-335.
8. Pundir, P. S., & Patawa, R. (2019). Stochastic behavior of dissimilar units cold standby system waiting for repair. *Life Cycle Reliability and Safety Engineering*, *8*, 43-53.
9. Pundir, P. S., Patawa, R., & Gupta, P. K. (2018). Stochastic outlook of two non-identical unit parallel system with priority in repair. *Cogent Mathematics & Statistics*, *5*(1), 1467208.

10. Rehmert, I. J., & Nachlas, J. A. (2008). Availability analysis for the quasi-renewal process. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, *39*(1), 272-280.
11. Shanker, R., Hagos, F., & Sujatha, S. (2016). On modeling of lifetime data using one parameter Akash, Lindley and exponential distributions. *Biometrics & Biostatistics International Journal*, *3*(2), 1-10.
12. Yadavalli, V. S. S., Botha, M., & Bekker, A. (2002). Asymptotic confidence limits for the steady state availability of a two-unit parallel system with'preparation time'for the repair facility. *Asia-Pacific Journal of Operational Research*, *19*(2), 249.
13. Yang, D. Y., & Tsao, C. L. (2019). Reliability and availability analysis of standby systems with working vacations and retrial of failed components. *Reliability Engineering & System Safety*, *182*, 46-55.